

Computer Science 1820

Solutions for Recommended Exercises

Section 6.3

$$2. P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E) \cdot P(F|E)}{P(F)} = \frac{(2/3) \cdot (5/8)}{3/4} = \frac{10/24}{3/4} = \frac{5}{12} \cdot \frac{4}{3} = \boxed{5/9.}$$

4. Let F be the event that Ann picks the *first* box, S be the event that she picks the *second* box, and O be the event that she selects an *orange* ball. Then, by Bayes' Theorem,

$$\begin{aligned} P(S|O) &= \frac{P(S) \cdot P(O|S)}{P(S) \cdot P(O|S) + P(\bar{S}) \cdot P(O|\bar{S})} = \frac{P(S) \cdot P(O|S)}{P(S) \cdot P(O|S) + P(F) \cdot P(O|F)} \\ &= \frac{(1/2) \cdot P(O|S)}{(1/2) \cdot P(O|S) + (1/2) \cdot P(O|F)} = \frac{(1/2) \cdot P(O|S)}{(1/2) \cdot P(O|S) + (1/2) \cdot (3/7)} \\ &= \frac{(1/2) \cdot (5/11)}{(1/2) \cdot (5/11) + (1/2) \cdot (3/7)} = \frac{(1/2) \cdot (5/11)}{(1/2) \cdot (5/11) + (1/2) \cdot (3/7)} \cdot \frac{2}{2} \\ &= \frac{5/11}{(5/11) + (3/7)} = \frac{5/11}{(35/77) + (33/77)} = \frac{5/11}{68/77} = \frac{5}{11} \cdot \frac{77}{68} = \boxed{35/68.} \end{aligned}$$

6. Let T be the event that a player *tests* positive for steroids and U be the event that the player *uses* steroids. Then, by Bayes' Theorem,

$$\begin{aligned} P(U|T) &= \frac{P(U) \cdot P(T|U)}{P(U) \cdot P(T|U) + P(\bar{U}) \cdot P(T|\bar{U})} = \frac{P(U) \cdot P(T|U)}{P(U) \cdot P(T|U) + (1 - P(U)) \cdot P(T|\bar{U})} \\ &= \frac{0.05 \cdot P(T|U)}{0.05 \cdot P(T|U) + (1 - 0.05) \cdot P(T|\bar{U})} = \frac{0.05 \cdot P(T|U)}{0.05 \cdot P(T|U) + 0.95 \cdot P(T|\bar{U})} \\ &= \frac{0.05 \cdot 0.98}{0.05 \cdot 0.98 + 0.95 \cdot P(T|\bar{U})} = \frac{0.05 \cdot 0.98}{0.05 \cdot 0.98 + 0.95 \cdot 0.12} = \frac{0.049}{0.049 + 0.114} = \frac{0.049}{0.163} \\ &= \boxed{49/163.} \end{aligned}$$