INSTRUCTIONS:

Write your answers on the exam booklet provided. Write your name on the exam booklet. Attempt all problems. Time: 50 min. Total: 50 pts.

Problem 1: **[total 20 pts]** You are given the coefficients $a_0, \ldots a_n$ of the degree n polynomial $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$. Your answer to the following questions should only use additions and multiplications.

a) **[6 pts]** Describe an algorithm to evaluate the polynomial given a value for x.

b) **[4 pts]** Estimate the efficiency class of your algorithm (identify function $f(n)$ so that the time complexity of your algorithm is $O(f(n))$). Explain. Is it correct to say that the efficiency class of your algorithm is $\Theta(f(n))$? Explain why or why not.

c) **[5 pts]** For the special case of evaluating the polynomial $p(x) = ax^n + b$, where a and b are known coefficients and $n = 2^k$ for some integer k, describe an algorithm with time complexity better than $O(n)$. Explain.

d) **[5 pts]** Extend your algorithm from subproblem (1c) to evaluate polynomials $p(x) = ax^n + b$ and handle any integer n, not just powers of two. Hint: exploit the binary expansion of exponent n in a way similar to the Russian peasant multiplication method; try to write a recursive algorithm, it may be easier.

Problem 2: **[10 pts]**

a) What is the worst case complexity of the Quicksort algorithm? Give an array with five elements illustrative of the worst case.

b) Suppose you have a routine to find the median value of an unsorted array of n elements in time $O(n)$. Can this routine be used to improve the worst case complexity of Quicksort? Explain why or why not.

Problem 3: **[total 10 pts]** Suppose you have a five element array $A[0 \ldots 4]$ sorted in non-decreasing order, with $A[0] = 1$ and $A[4] = 5$. We search for key $K = 2$.

a) **[2 pts]** The first comparison in binary search involves K and some element $A[i]$. What is the value of i?

b) **[2 pts]** The fist comparison in interpolation search involves K and some element $A[j]$. What is the value of j? Hint: to answer the question, just draw the linear function interpolating the values of the array.

c) **[3 pts]** List a possible set of values for elements $A[1], A[2], A[3]$ so that binary search terminates immediately after the comparisons between K and $A[i]$ (no other element from the array is accessed for comparisons with K).

d) **[3 pts]** Is it possible to assign values for elements $A[1], A[2], A[3]$ so that interpolation search terminates immediately after the comparisons between K and $A[j]$ (no other element from the array is accessed for comparisons with K), AND K is NOT present in the array? Explain.

Problem 4: **[10 pts]** Suppose you are choosing between the following three algorithms:

A: the algorithm divides a problem of size n into five subproblems of size $\frac{n}{2}$, recursively solves each subproblem, and combines the solutions in linear time.

B: the algorithm solves a problem of size n by recursively solving two subproblems of size $n-1$, and combining the solution in constant time.

C: the algorithm solves a problem of size n by dividing it into nine subproblems of size $\frac{n}{3}$, recursively solving each subproblem, and combining the solutions in $O(n^2)$ time.

What is the running time of each of these algorithms in O-notation, and which one would you chose?

Recall the Master theorem: if $T(n) = aT(n/b) + n^d$, then

$$T(n) \in \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d. \end{cases}$$