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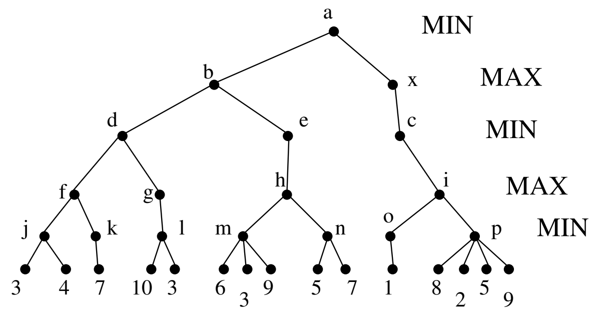
| CPSC 3750 – A.I. | **Due Mar. 24** (in class) |
| Assignment 3 | Total marks: 45 |

1) **(5 pts)** Assign minimax values to all nodes in the following game tree, then describe the optimal play sequence by listing the corresponding sequence of state labels.




Solution:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| p | o | n | m | l | k | j | i |
| 2 | 1 | 5 | 3 | 3 | 7 | 3 | 2 |
| h | g | f | c | e | d | x | b |
| 5 | 3 | 7 | 2 | 5 | 3 | 2 | 5 |
| a | | | | | | | |
| 2 | | | | | | | |

The optimal play follows states: a x c i p 2.

2) **(20 pts)** Let's call a player perfect if it never makes mistakes when playing. For example, if MAX is perfect, then it always chooses the move that maximizes the utility (minimax trees are built assuming perfect players).

Prove or disprove the following statements:

(a) Let $\alpha$ be the utility obtained when two perfect players play. Assume now that the game is repeated but the MAX player is imperfect (MAX makes mistakes in that it doesn't choose the move with maximum utility). Then, the new utility obtained is equal or smaller than $\alpha$.

Solution:

True. Assume MAX makes one mistake at a state $s$, in that it chooses a move towards state $s'$. This means that the utility of $s'$ is smaller or equal than the optimal choice. Therefore, if both MAX and MIN play optimally from now on, the play ends with the utility equal to that of $s'$.

(b) Like in the previous statement, let $\alpha$ be the utility of a game when both players are perfect. If the game is repeated, MAX plays first and BOTH players make mistakes, then the utility obtained is equal or larger than $\alpha$.

Solution:

False. If both player make mistakes, then the sequence of moves describes an arbitrary path in the minimax tree and any leaf can be reached. Therefore, the utility obtained can be larger, equal, or smaller than the utility of the optimal play.

3) **(10 pts)** Consider a vocabulary with 3 symbols: $A$, $B$, $C$. How many models are there for the following sentences:

(a) $(A \wedge B) \vee (B \wedge C)$

(b) $A \vee B$

(c) $A \wedge B \wedge C$

(d) $A \Leftrightarrow B \Leftrightarrow C$

Solution:

We need to count the number of models (values for $A$, $B$, and $C$ for which the sentences are true.

(a) : the sentence is 1 in 3 models:

| $A$ | $B$ | $C$ |
|---|---|---|
| 1 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

(b): There are 6 models. * means both 0 and 1 values are valid.

| $A$ | $B$ | $C$ |
|---|---|---|
| 1 | 0 | * |
| 0 | 1 | * |
| 1 | 1 | * |

(c): One model.

| $A$ | $B$ | $C$ |
|---|---|---|
| 1 | 1 | 1 |

(d): $A \Leftrightarrow B$ is 1 when $A$ equals $B$. So, when $A$ and $B$ are equal, $C$ should be 1, and when $A$ and $B$ differ, $C$ should be 0 in order for the sentence to be 1. Thus, there are 4 models:

| $A$ | $B$ | $C$ |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

4) **(10 pts)** Consider the minesweeper computer game (`http://en.wikipedia.org/wiki/Minesweeper_(computer_game)`). Let $X_{ij}$ be true if square $(i, j)$ has a mine. Write down a sentence in propositional logic that asserts that there are exactly two mines adjacent to square $(1, 1)$ where $(1, 1)$ is the bottom left corner of the board. Then generalize the sentence to assert that $k$ of $n$ neighbours contain mines.

Solution:

There are three adjacent squares to $(1, 1)$. The sentence contains all combinations of two positive symbols with the remaining symbols negative. This is $(X_{21} \wedge \neg X_{22} \wedge X_{21}) \vee (X_{21} \wedge X_{22} \wedge \neg X_{21}) \vee (\neg X_{21} \wedge X_{22} \wedge X_{21})$.