### 2: Elementary mathematical notions

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### Vector spaces

2 Combinations of vectors

- Linear hull
- Affine hull
- Convex hull
- Positive cone

- Decision variables:  $x_1, x_2, \dots x_n$
- Onstraints:

 $f_j(x_1,\ldots,x_n) \leq \gamma_j,$  $g_h(x_1,\ldots,x_n) = \delta_h$ 

Objective function:  $z(x_1, \ldots x_n)$ 

### Mathematical representation

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#### Mathematical representation

Real vector spaces (with inner product),  $\mathbb{R}^n$ :

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#### Mathematical representation

Real vector spaces (with inner product),  $\mathbb{R}^n$ :

- decision variables: vector.
- scalar: the reals
- operations:
  - Vector addition
  - Scalar-vector multiplication.
  - Closed under vector addition and scalar-vector multiplication.

(see course external resources)

### (Matrix) Notation



- Column vectors: *a*,...*z*.
- Row vectors:  $a^T, \ldots z^T$ .
- Matrices: *A*,...*Z*.
- Scalars:  $\alpha, \ldots \omega$ .

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### Linear Hulls (subspaces)



Subspace S of  $\mathbb{R}^n$ : a vector spaces with S closed over vector + and scalar  $\cdot$ . (0  $\in$  S).

 $\boldsymbol{b} = \alpha_1 \boldsymbol{x}_1 + \alpha_2 \boldsymbol{x}_2 + \ldots + \alpha_k \boldsymbol{x}_k.$ 

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### Affine Hulls



#### $\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_k x_k = b$ $\alpha_1 + \alpha_2 + \ldots + \alpha_k = 1$

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### **Convex Hulls**



$$\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_k x_k = b$$
  

$$\alpha_1 + \alpha_2 + \ldots + \alpha_k = 1$$
  

$$\alpha_1, \ldots, \alpha_k > 0$$

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### Positive cone





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