Math Review

- Matrices, especially square matrices
- Matrix transpose
- Identity matrix
- Matrix addition, subtraction, product

Matrix Properties

- $\mathbf{A} \cdot \mathbf{I} = \mathbf{I} \cdot \mathbf{A} = \mathbf{A}$
- $(\mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \cdot \mathbf{C})$
- Generally $\mathbf{A} \cdot \mathbf{B} \neq \mathbf{B} \cdot \mathbf{A}$
- $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{C} + \mathbf{B} \cdot \mathbf{C}$
- $(\mathbf{A} + \mathbf{B})^t = \mathbf{A}^t + \mathbf{B}^t$
- $(\mathbf{A} \cdot \mathbf{B})^t = \mathbf{B}^t \cdot \mathbf{A}^t$

Matrix Properties

- If $\mathbf{A}$ is square and invertible, there exists an inverse $\mathbf{A}^{-1}$ such that $\mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{A}^{-1} \cdot \mathbf{A} = \mathbf{I}$.
- Not all square matrices are invertible
- $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
- A square matrix is invertible iff its determinant is nonzero.
- If $\mathbf{A}$ transforms an object (e.g. scaling, translation, etc.), then $\mathbf{A}^{-1}$ takes the transformed object back to the original.

Vectors

- A vector can be repsented as an $n \times 1$ matrix (column vector) or a $1 \times n$ matrix (row vector)
- Dot product of two vectors of the same size: $\vec{v} \cdot \vec{w}$
- Cross product of two 2D or 3D vector: $\vec{v} \times \vec{w}$
- A vector can be represented as a linear combination of the standard basis unit vectors
- You can add and subtract vectors, multiply them by a scalar
- Vector can be used to specify a location (point), or a direction and length.
- Length of vector $\|\vec{v}\|$

Basic Trigonometry

- Sin and Cosine
- Polar coordinates
- Degrees vs. radians

Geometric Objects

- Geometric objects are specified by vertices
- e.g. A line or a line segment is specified by 2 distinct vertices
- A triangle is specified by 3 distinct vertices

Viewers

- Need to specify where the camera is, what direction it is pointing at, etc.
- Need to specify lighting sources (single, multiple, etc.)

Geometric Primitives

- Point:
  - a location in space (x, y) or (x, y, z)
  - no size or shape
  - Coordinate system (left-hand vs. right-hand)
- Scalar: a real number
- Vector:
  - length and direction
  - no fixed starting point
- Vectors and points are both represented as a tuple, and sometimes are used at the same time.

Lines (2D)

- A line is specified by two distinct points
- Extends infinitely
- Implicit equation: $ax + by + c = 0$
- Do not use $y = mx + b$ (why not?)
- Parametric equation: $(x, y) = (x_0, y_0) + t(d_x, d_y)$
- Another form of parametric equation:

$$(x, y) = t(x_0, y_0) + (1 - t)(x_1, y_1)$$

- $t = 0$ or $t = 1$ are the two points
- $t \in [0, 1]$: between the two points

Line Segments (2D)

- Specified by 2 distinct end points.
- Parametric form: $t \in [0, 1]$.
- Ray: half infinite. $t \geq 0$
- How do we intersect lines? Line segments?

Line Segment Intersection (2D)

- Solve system of 2 equations of 2 lines (implicit or parametric): may have no (parallel) or infinite (coincident) intersections.
- Need to ensure intersection points are on line segment : parametric form easiest

Lines (3D)

- Almost the same as 2D lines, except that each coordinate has 3 components (x, y, z)
- Parametric form is easiest: $(x, y, z) = (x_0, y_0, z_0) + t(d_x, d_y, d_z)$
- For line segments $t \in [0, 1]$
- Implicit form does not give a line: it gives a plane.

Planes (3D)

- Implicit form:

$$ax + by + cz + d = 0$$

  The vector (a, b, c) is the normal vector to the plane

- Defined by 3 non-colinear points p_1, p_2, p_3
- Equation obtained by solving a system of 3 equations in 4 unknowns (infinite number of solutions)
- Can also obtain normal vector with cross product of $(p_2 - p_1) \times (p_3 - p_1)$

Triangles (2D/3D)

- Defined by three non-colinear points.
- More difficult to define coordinates inside the triangle.
- One method is to use barycentric coordinates: $p = \lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3$ where $\lambda_1 + \lambda_2 + \lambda_3 = 1$