Experiments in Simple One-Dimensional Lossy Image Compression Schemes*

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Abstract

This paper examines some one-dimensional lossy image compression schemes. To perform one-dimensional compression, an image is scanned into a one-dimensional array which is then compressed. Three types of scanning are considered: raster scan, Hilbert scan, and binary scan. Hilbert and binary scan are found to outperform raster scan. The methods considered for compression are integer wavelet transforms, a piecewise approximation with triggers (PAT) procedure introduced by Walach and Karnin [6], and a modified PAT algorithm (MPAT). Wavelets outperform MPAT in compression, and MPAT in turn outperforms PAT; however, MPAT and PAT are computationally far simpler than wavelet transforms.

1. Introduction

Recently, digital images have been exploding in usage and in size. Many algorithms perform well in terms of compression, but they are often computationally intensive. This paper examines one-dimensional schemes which are simpler than two-dimensional methods. They will naturally not compress images as well as two-dimensional methods. However, they operate at such low complexity that they can be implemented on systems with little processing power, notably portable units.

2. Scanning Methods

Three methods were implemented in our experiments to convert the two-dimensional image into a one-dimensional array: raster scan, Hilbert scan, and binary scan. They are all invertible mappings from a greyscale image to a one-dimensional array. A scanning method should preserve the original correlation between neighbouring pixels. This will help the compression algorithm remove local redundancies in the array. The scanning algorithms studied are computationally simple.

Raster scanning preserves horizontal correlations well but vertical correlations are lost. Hilbert scanning uses a space-filling curve known as the Hilbert curve [3], preserves local correlation well [5], but is restricted to images of dimensions $2^n \times 2^n$. On the other hand, binary scanning [1] works on arbitrary rectangles and preserves correlations similar to Hilbert scanning.

3. Compression Methods

Once the image has been scanned into a one-dimensional array, various methods can be used to compress the data. The compressed output is entropy coded using an arithmetic coder with multiple models—a different model for each type of data. The use of arithmetic coders does not significantly increase the complexity.

One popular way of compressing data is to use wavelet transforms to extract important information. This was implemented in our experiments to allow comparisons to be made between the effectiveness of wavelet based compression and PAT based algorithms on one-dimensional signals. For the experiments, a ten-level pyramid of integer S+P transforms was selected. The method chosen to compress the output from the wavelet filters was to encode the output one bit-plane at a time, taking advantage of the long runs of zeros. The encoding process stops when the desired bit rate is reached.

PAT offers good compression at very low complexity levels. The concept is proposed by Walach and Karnin [6]. The original idea is to "lay" a yardstick down on a one-dimensional signal and record the horizontal distance it travels. This is generalized using a trigger function TF, so that the transmitted distance becomes $\min\{i, i_{max} : |s_{x+i} - r_x| > TF(i)\}$ where i_{max} is the maximum length, s_x is the signal value at x, and the reconstruction at x is r_x. The sign of $s_{x+i} - r_x$ is transmitted after each distance is sent (called a trigger).

A simple modification to the above procedure is suggested by Walach and Karnin, which is the use of thresholds. When the distance to be transmitted is one and $|r_x - s_{x+1}| > TF(0)$, then it is likely that the signal is near an edge. In this event, a zero is transmitted followed

*This research is supported in part by the Motorola Wireless Data Group and the Canadian Natural Sciences and Engineering Research Council under Grant OGP9198.
†Supported by the SCP '96 scholarship.
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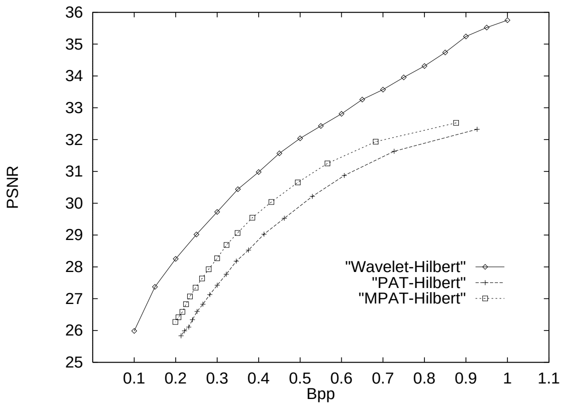

Fig. 1. Comparison of compression methods using the Hilbert scan on Lena (512×512).

by a three-bit (eight level) quantized value. Using different models for distances, signs and quantized values, the output is compressed with an arithmetic coder. The original paper suggests the use of line subsampling to increase performance, but that introduces significant visual distortion; hence it is not used in these experiments. For the decompression phase, the signal is linearly interpolated between the reconstructed values.

Some changes can be made to the original PAT algorithm to improve its numerical results and to add flexibility to the algorithm, creating MPAT. Also, a fixed trigger function gives a file size that cannot be well predicted. Extra parameters that can alter the file size and/or distortion are useful for adding flexibility to PAT. Three significant changes made to PAT are described below.

Trigger Function: Earlier papers on PAT use the trigger function originally suggested in [6], In these experiments a wide variety of trigger functions are examined from the set of functions $TF(x) = ae^{bx} + c$ where a, b, and c are parameters. In addition, i_{max} is set to 64.

Context Modeling: Simple context modeling is introduced so that the model used to encode the current distance is based on the last transmitted distance. This choice can be made by a table lookup, which incurs negligible computational overhead. Context modeling is further increased by coding the current sign based on the previous sign.

Quadratic Interpolation: Another modification is to use quadratic interpolation instead of linear interpolation in the reconstruction. Sharp changes are eroded under linear interpolation, but are better preserved by quadratic interpolation.

4. Performance Evaluation

Some results are now presented for the PAT based algorithms. From the set of functions classified by $TF(x) = ae^{bx} + c$, the members from the subset $TF(x) = ae^{-0.05x} + 2$ were found to be near optimal on rate-distortion graphs. The results from this set of trigger functions are quite good, and they are better than those given in [4]. Compared to the number of triggers generated, the thresholds occur infrequently. The average distance that a trigger covers is approximately 13. The Hilbert and binary scans perform almost identically. Both scans have rate-distortion curves that are well above the simple raster scan. This confirms the value of a good scanning method for one-dimensional compression. All three compression schemes are compared in Figure 1. It is clear that wavelet based compression performs better than the PAT based algorithms. This gain is achieved at the expense of a higher computational cost than the PAT based algorithms. It can also be seen from the graphs that MPAT performs consistently better than PAT. This advantage seems to diminish with increasing bit rates, but methods are under study to improve performance at higher bit rates. Visually, the PAT methods avoid the “blurry” and “blocky” problems of traditional compression schemes at low PSNR values.

The complexity of PAT based algorithms is extremely low. An operation count was performed on the compression schemes. For wavelet based compression, only the operations used during the wavelet transformations were included. While the wavelet transforms required 7.99 additions per pixel, the PAT based algorithms required only 1.23. The benefits of PAT based algorithms can also be seen in that it is simpler than quadtree algorithms and quadtree algorithms are among the most computationally efficient of two-dimensional methods according to [2]. From our experiments, we can say that MPAT and PAT are an order of magnitude simpler than two-dimensional analogues.

5. Conclusion

It is clear that the PAT based algorithms perform well as their performance is comparable to the performance of wavelet based compression. It is also evident that a good scanning method is crucial in improving compression. Using binary scan, these results are applicable to images of arbitrary dimensions. The MPAT algorithm consistently outperforms PAT. The greatest benefit of PAT based algorithms is an extremely low computational requirement.

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