$$X \subset Y \quad \Leftrightarrow \quad \forall x \in X, x \in Y$$

The meaning of this sentence guides how we can prove it (for all introduction) and how we can use it when we know it already (for all elimination).

Suppose A, B, and C are sets. Prove the deduction:

$$B \subset C, \qquad \therefore (A \cap B) \subset (A \cap C)$$

$$X = \emptyset \quad \Leftrightarrow \quad \neg \exists x \in \mathcal{U}, x \in X \quad \Leftrightarrow \quad \forall x \in \mathcal{U}, x \notin X$$

$$X \neq \emptyset \quad \Leftrightarrow \quad \exists x \in \mathcal{U}, x \in X$$

The meaning of these sentences guides how we can prove them and how we can use them when we know them already.

Suppose A, B, and C are sets. Prove the deduction:

$$A \cap B = \emptyset, \quad C \subset B, \quad \therefore A \cap C = \emptyset$$

$$X = Y \iff ((X \subset Y) \,\&\, (Y \subset X))$$

The meaning of these sentences guides how we can prove them and how we can use them when we know them already.

Suppose A, B, and C are sets. Prove the deduction:

$$\therefore (A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

Suppose that A, B, and C are sets. Show that the deduction:

$$C \subset A \cup B, \qquad \therefore (C \subset A) \vee (C \subset B)$$

is **not** valid.

Suppose A, B, X, and Y are sets. Prove the deduction:

$$(X \subset A), \qquad (Y \subset B), \quad \therefore X \cap Y \subset A \cap B$$

Suppose A, B, and C are sets. Prove the deduction:

$$A \subset B, \qquad \therefore C \setminus B \subset C \setminus A$$

Suppose A, B, and C are sets. Show that the deduction:

$$A \cap B = \emptyset, \quad B \cap C = \emptyset, \quad \therefore A \cap C = \emptyset$$

is **not** valid.