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Polar Coordinates

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Overview

Warm-Up

Find the area encolosed by the astroid
$x = \cos^3(t), y = \sin^3(t), t \in [0, 2\pi]$. How about its length?

The polar coordinate system

Polar coordinates are defined in terms of the usual Cartesian plane. But instead of using x and y to locate a point, we use r and θ.

- r: distance from the origin to the point. (Defines a circle)
- θ: direction of the point. (Defines a point on the circle)

Converting

Example

Convert the following points from rectangular to polar coordinates:

1. $(-4, 0)$
2. $(3, -3)$
3. $(0, 0)$

Convert the following points from polar to rectangular coordinates:

1. $(3, \pi/3)$
2. $(-2, \pi/4)$
3. $(1, -5\pi/6)$

Simple polar curves

Some curves are really simple in polar coordinates. For example:

1. $r = 3$
2. $\theta = 2\pi/3$
3. $r = \theta$

Simple rectangular curves

Some curves are simple in rectangular coordinates but not so great in polar:

1. $y = 3x + 2$
2. $y = 3$
3. $y = x^2$

Polar curves: limaçons

A typical polar curve has the form $r = f(\theta)$. We can generate a lot of interesting curves this way.

Example

Describe (sketch) the polar curve:

1. $r = 1 + \cos\theta$
2. $r = 2 + \cos\theta$
3. $r = 1 + 2\sin\theta$

Polar curves: roses

Describe (sketch) the polar curve:

1. $r = \sin(2\theta)$
2. $r = \cos(4\theta)$

Tangent lines

Polar curves are really parametric curves: $x = r\cos\theta$ and $y = r\sin\theta$, so $r = f(\theta)$ defines $x(\theta), y(\theta)$. That gives $\dfrac{dy}{dx} = \dfrac{y'(\theta)}{x'(\theta)}$.

Example

Find the equation of the tangent line to $r = \cos(3\theta)$ when $\theta = \pi/4$.

Example

1. Determine the location of the horizontal and vertical tangents of the limaçons $r = 1 + 2\cos\theta$.
2. Find the equations of the tangent lines to the curve at the origin. (There are two!)

Area

The polar area formula is given by $A = \int_\alpha^\beta \frac{1}{2} r^2\,d\theta$.

Example

Find the area enclosed by one leaf of the curve $r = \sin(3\theta)$.

Example

Find the area that lies inside the outer loop of the limaçon $r = 1 + 2\cos\theta$, but outside the inner loop.

Example

Find the area that lies inside the curve $r = 4\cos\theta$ but outside the curve $r = 4\sin\theta$

Arc length

(If we have time.)