

(b) Let 
$$S = span\{(1, 1, 1, 1), (1, 1, -1, 1), (1, -1, 1, 1)\}$$
. (7) Using part (a), determine if the vector  $(-2, 4, 0, -2)$  is in  $S$ .

(c) (3) Is it true that 
$$S = \mathbb{R}^4$$
? Explain

2. (a) (b) Verify that the three vectors 
$$\underline{\mathbf{u}} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), \ \underline{\mathbf{v}} = (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), \ \underline{\mathbf{w}} = (\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$$
 form an orthonormal basis for  $\mathbb{R}^3$ .

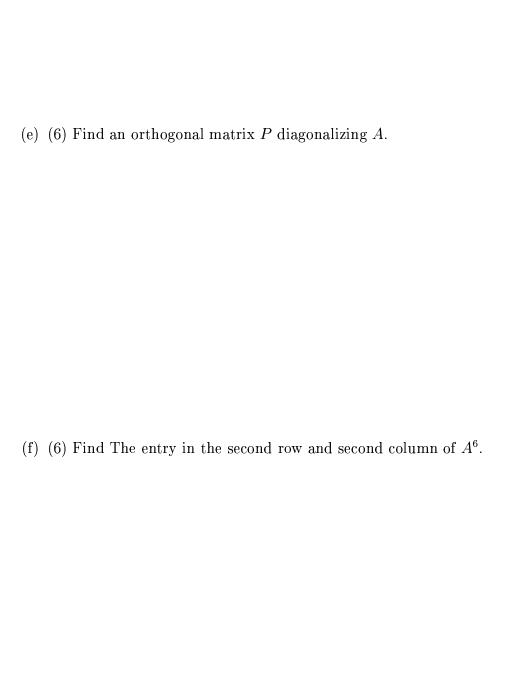
(b) (4) Is it possible to find a vector which is not in the span of the vectors 
$$\underline{\mathbf{u}}$$
,  $\underline{\mathbf{v}}$  and  $\underline{\mathbf{w}}$ ?

3. Let

$$A = \left[ \begin{array}{ccc} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{array} \right].$$

- (a) (4) show that (1,1,1) is an eigenvector of A and find the corresponding eigenvalue.
- (b) (4) show that (1,-1,0) is an eigenvector of A and find the corresponding eigenvalue.
- (c) (6) Find all the eigenvalues of A.

(d) (10) Find a basis for each of the eigenspaces of the matrix A.



4. (5,5,5) Which of the following subsets is a subspace of  $\mathbb{R}^3$ ? You need to justify your claim.

(a) 
$$\{(x, y, z): x + 3y - z = 5\}$$

(b) 
$$\{(x, y, z): x + 3y - z = 0\}$$

(c) 
$$\{(x, y, z) : xyz = 0\}$$

- 5. Given  $S = \{(-1, 0, 1), (-2, 0, 1)\}.$ 
  - (a) (4) Describe the vectors in the span of S.
  - (b) (7) Determine whether the vector  $\underline{\mathbf{v}} = (1, 0, 4)$  is in the span of S and if the vector is in the span, write it as a linear combination of elements of S

6. Let 
$$A = \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 3 & 3 & 1 \end{bmatrix}$$
.

(a) (5) Find the dimension of the row space of A.

(b) (8) Find a basis and the dimension of the solution set of the equation  $A\underline{\mathbf{x}} = 0$ .