1. (a) (3) Write the augmented matrix for the following system of linear equations.

$$\begin{cases} x - 3y + z + t + 2u = 4 \\ 2x - 6y + z + 2t + u = 4 \\ 2x - 5y + 2z + 2t - u = -1 \end{cases}$$

(b) (10) Solve the system.

(c) (3) Check your answer.

2. (5 each part) If A, B, and C are 2×2 matrices, prove or disprove the following:

   (a) $AB = BA$ for all A and B.

   (b) If $A^2 = 0$ and A is invertible, then $A = 0$.

   (c) If $AB = AC$ and A is invertible, then $B = C$ for all B and C.

   (d) If $AB = BA$, then $A^2 - B^2 = (A + B)(A - B)$.

3. (a) (8) Find elementary matrices $E_1, E_2, \cdots, E_n$, such that $E_n \cdots E_2 E_1 A$ is the reduced echelon form of A, where $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$

   (b) (6) Find $E_n \cdots E_2 E_1$ and check your answer.

4. (a) (6) Evaluate the determinant of the matrix $\begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & c & 1 & 1 \\ 0 & 0 & a & b \\ 0 & 0 & b & a \end{pmatrix}$.

(b) (3) For what values of a, b, and c is the matrix singular

5. (a) (8) Let $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{pmatrix}$, then (fill in the four missing entries) $A^{-1} = \frac{1}{20}\begin{pmatrix} -2 & & -2 \\ & -13 & \\ 5 & & -5 \end{pmatrix}$.
(Use cofactor method)

(b) (3) Check your answer.

(c) (6) Write the systems of equations $\begin{cases} x + 3y + 5z^2 = 1 \\ 2x + y + z^2 = 1 \\ 3x + 4y + 2z^2 = 1 \end{cases}$
in the matrix form $AX = B$ and use part (a) to solve it.

6. (a) (7) Find all values of a for which the determinant of the coefficients in the system of equations

$$\begin{cases} x + 2y + z = 1 \\ x + ay + 2z = 1 \\ 2x + 4y + az = 1 \end{cases}$$

is NOT zero.

(b) (9) Let $a = 1$. Use Cramer's rule to solve the system of equations.

7. (a) (4) Suppose that A, B, C are square matrices of the same order, $AC = I$ and $BA = I$. Show that $B = C$.

(b) (4) Let A and B be two invertible matrices of the same order. Show that $(AB)^{-1} = B^{-1}A^{-1}$.