1. Find the following: Do not simplify your answer.

[5] (a) y' if $y = \displaystyle\int_{\sin x}^{x^2} \frac{t}{2t^2+1}\,dt$.

[5] (b) $f''(x)$, where $f(x) = \displaystyle\int_2^1 \left(\int_2^3 (t^2+10)\,dt\right) du$.

[5] (c) $f'(0)$, where $f(x) = \cos\big|(|x-1|-x)\big|$.

[5] (d) All points at which the function $f(x) = \cos\big|(|x-1|-x)\big|$ may not be differentiable.

2. Evaluate the following integrals:

[7] (a) $\displaystyle\int x^5\, \sqrt[3]{x^3-3}\,dx$.

[7] (b) $\displaystyle\int_0^4 \left(\frac{(x^2+3)^2}{\sqrt{x^2+9}}\right)' dx$.

[6] (c) $\displaystyle\int \frac{\cos^3(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx$.

[10] 3. (a) An open cylindrical can is made to contain $125\pi\ cm^3$ of liquid. Find the minimum cost to make the can, if the cost to make the can is .04¢ per cm^2. (The volume v of a cylinder with height h and radius of base r is $v = \pi r^2 h$ and the lateral surface area is $A = 2\pi r h$)

[8] (b) Use the *definition of integral* to evaluate $\displaystyle\int_0^2 (3x^2-4x)\,dx$. (Hint: $\sum\limits_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum\limits_{i=1}^{n} i^2 = \frac{n(n+\frac{1}{2})(n+1)}{3}$, $\sum\limits_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$)

4. Let $y = \dfrac{9(3-x^2)}{x^3}$ then $y' = \dfrac{9(-9+x^2)}{x^4}$ and $y'' = \dfrac{18(18-x^2)}{x^5}$.

[5] (a) Find $\lim\limits_{x\to 0+} \dfrac{9(3-x^2)}{x^3}$, $\lim\limits_{x\to 0-} \dfrac{9(3-x^2)}{x^3}$, $\lim\limits_{x\to\infty} \dfrac{9(3-x^2)}{x^3}$, $\lim\limits_{x\to-\infty} \dfrac{9(3-x^2)}{x^3}$ and all asymptotes, if any.

[4] (b) Do a complete first derivative analysis.

[3] (c) Do a complete second derivative analysis.

[3] (d) Give a careful sketch of the graph of the function.

[5] (e) Sketch a graph of the function: $y = \dfrac{\frac{9(3-x^2)}{x^3}}{\left|\frac{9(3-x^2)}{x^3}\right|}$, using part (d) above.

[5] 5. (a) Let $y = f(x)$ be a function for which $f''(x)$ is continuous on the interval $[1,2]$. If $f'(1) = f'(2)$, evaluate the integral, $\int_1^2 \big(f(x)\big)''\,dx$.

[5] (b) Find $\lim\limits_{x\to-\infty} \left(3x + \sqrt{9x^2+9x}\right)$.

[5] (c) Show that the function $f(x) = \displaystyle\int_{-1}^{x} \frac{1}{|t|}\, dt$, $x < 0$, is concave up on the interval $(-\infty, 0)$.

[7] 6. Prove the first part of the Fundamental Theorem of Calculus:
Let $y = f(x)$ be a continuous function on the interval $[a, b]$.
Show that $(\int_a^x f(t)\, dt)' = f(x)$, for all x in (a, b).