On the Siamese Twin Designs

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Abstract. Let $4n^2$ be the order of a Bush-type Hadamard matrix with $q = (2n+1)^2$ a prime power. It is shown that there is a weighing matrix

$$W(4(q^m + q^{m-1} + \cdots + q + 1)n^2, 4q^m n^2)$$

which can be used to construct a pair of symmetric designs with the parameters

$$\nu = 4(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa = q^m(2n^2 + n), \quad \lambda = q^m(n^2 + n)$$

for every positive integer m. As a corollary we get a new class of symmetric designs with parameters

$$\nu = 16(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa = q^m(8n^2 + 2n), \quad \lambda = q^m(4n^2 + 2n)$$

for all positive integers m and n, where $4n$ is the order a Hadamard matrix.

1 Introduction

A Bush-type Hadamard matrix is a block matrix $H = [H_{ij}]$ of order $4n^2$ with block size $2n$, $H_{ii} = J_{2n}$ and $H_{ij}J_{2n} = J_{2n}H_{ij} = 0$, $i \neq j$, $1 \leq i \leq 2n$, $1 \leq j \leq 2n$, where J_{2n} is the $2n$ by $2n$ matrix of all entries 1. It is known that symmetric Bush–type Hadamard matrices exist for all orders $16n^2$ for all values of n for which there is a Hadamard matrix of order $4n$, see [8] for details.

A balanced generalized weighing matrix $BGW(\nu,\kappa,\lambda)$ over a group G is a matrix $W = [w_{ij}]$ of order ν , with $w_{ij} \in G \cup \{0\}$ such that each row and column of W has κ non–zero entries and for each $k \neq l$, the multiset $\{w_{kj}w_{lj}^{-1} : 1 \leq j \leq \nu, w_{kj} \neq 0, w_{lj} \neq 0\}$ contains $\lambda/|G|$ copies of every element of G.

In [7] the author used a Bush–type Hadamard matrix of order $4n^2$, with $q = (2n-1)^2$ a prime power, in a class of balanced generalized weighing matrices over a cyclic group to construct a new class of symmetric designs with parameters $\nu = 4(q^m + q^{m-1} + \cdots + q + 1)n^2, \kappa = q^m(2n^2 - n), \lambda = q^m(n^2 - n)$, for every positive integers m. These symmetric designs were shown to be generated in pairs by equating certain entries of a weighing matrix to zero. The first class of symmetric designs with these parameters was first constructed by Ionin in [6] for $n = 3$. More precisely, Ionin [6] used a very special regular Hadamard matrix of order 36 in a class of balanced generalized weighing matrices $BGW(q^m + q^{m-1} + \cdots + q + 1, q^m, q^m - q^{m-1})$

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over a cyclic group of order t, where q is a prime power, m is a positive integer and t is a divisor of $q-1$, to construct his symmetric designs.

In this paper we again use a Bush–type Hadamard matrix of order $4n^2$, but this time with $q=(2n+1)^2$ a prime power to construct the sister class of symmetric designs with parameters $\nu = 4(q^m+q^{m-1}+\cdots+q+1)n^2$, $\kappa = q^m(2n^2+n)$, $\lambda = q^m(n^2+n)$ for every positive integer m. We will show that these designs again are generated in pairs, but unlike the designs in [7], the blocks of the designs here have some "common intersections". We use the term "Siamese twin design" to draw the attention to this property.

The problem of investigating the existence of Bush–type Hadamard matrices of order $4n^2$, n an odd integer, is a tough one. It is quite interesting to note that if such matrices exist, then the construction method given in this paper simplifies Ionin's method significantly. Moreover, the method used in this paper produces an infinite class of new symmetric designs from any Bush–type Hadamard matrix of new order $4n^2$, n odd such that the number $(2n+1)^2$ is a prime power, see [5]. On the other hand, the nonexistence of a symmetric Bush–type Hadamard matrix of order $4n^2$ would imply the nonexistence of projective plane of order $2n$. The nonexistence of a symmetric Bush–type Hadamard matrix of order 36 has been established in [2], though in a completely different setting. The smallest order for which the existence of Bush–type Hadamard matrices is unknown is $196 = 14^2$. We conjecture here, as in [7], that Bush-type Hadamard matrices exist for all orders $4n^2$, n an odd integer.

For a $(0,\pm 1)$-matrix K, let $K = K^+ - K^-$, where K^+ and K^- are $(0,1)$-matrices. The Kronecker product of two matrices $A=[a_{ij}]$ and B, denoted $A\otimes B$, is defined, as usual, by $A\otimes B = [a_{ij}B]$. For a matrix $A=[a_{ij}]$, denote by $|A|$ the matrix $[|a_{ij}|]$. Throughout the paper, $-$ represents -1.

2 Bush-type Hadamard matrices and Siamese twin designs

K. A. Bush [1] proved that if there exists a projective plane of order $2n$, then there is a Hadamard matrix H of order $4n^2$, such that:

1. H is symmetric.
2. $H=[H_{ij}]$, where H_{ij} are blocks of order $2n$, $H_{ii}=J_{2n}$ and $H_{ij}J_{2n}=J_{2n}H_{ij}=0$, for $i\neq j$, $1\leq i\leq 2n$, $1\leq j\leq 2n$.

Bush's interest was mainly in the nonexistence of such matrices. Whereas there are different methods to construct matrices of order $16n^2$ of the above type, (see [8]), we are not aware of nonexistence of matrices of this form of order $4n^2$ for any odd value of n, $n>3$.

Please note that in this paper by a *Bush-type Hadamard matrix* we mean an Hadamard matrix satisfying only condition 2 above, as we do not need to assume that H is symmetric for our construction. For completeness we include the following result of the author [8].

Theorem 1. *Let $4n$ be the order of a Hadamard matrix, then there is a Bush-type Hadamard matrix of order $16n^2$.*

Proof. Let K be a normalized Hadamard matrix of order $4n$. Let r_1, r_2, …, r_{4n} be the row vectors of K. Let $C_i = r_i^t r_i$, $i = 1,2,\ldots,4n$. Then it is easy to see that:

1. $C_i^t = C_i$, for $i = 1,2,\ldots,4n$.
2. $C_1 = J_{4n}, C_i J_{4n} = J_{4n} C_i = 0$, for $i = 2,\ldots,4n$.
3. $C_i C_j^t = 0$, for $i \neq j$, $1 \leq i, j \leq 4n$.
4. $\sum_{i=1}^{4n} C_i C_i^t = 16n^2 I_{4n}$.

Now let $H = \mathrm{circ}(C_1, C_2, \ldots, C_{4n})$, the block circulant matrix with first row $C_1 C_2 \ldots C_{4n}$. Then H is a Bush-type Hadamard matrix of order $16n^2$.

Example 2. Let

$$K = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & - & - \\ 1 & - & 1 & - \\ 1 & - & - & 1 \end{pmatrix}$$

Then,

$$r_1 = \begin{pmatrix} 1 \; 1 \; 1 \; 1 \end{pmatrix}$$

$$r_2 = \begin{pmatrix} 1 \; 1 \; - \; - \end{pmatrix}$$

$$r_3 = \begin{pmatrix} 1 \; - \; 1 \; - \end{pmatrix}$$

$$r_4 = \begin{pmatrix} 1 \; - \; - \; 1 \end{pmatrix}$$

$$C_1 = r_1^t r_1 = \begin{pmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \end{pmatrix}$$

$$C_2 = r_2^t r_2 = \begin{pmatrix} 1 & 1 & - & - \\ 1 & 1 & - & - \\ - & - & 1 & 1 \\ - & - & 1 & 1 \end{pmatrix}$$

$$C_3 = r_3^t r_3 = \begin{pmatrix} 1 & - & 1 & - \\ - & 1 & - & 1 \\ 1 & - & 1 & - \\ - & 1 & - & 1 \end{pmatrix}$$

$$C_4 = r_4^t r_4 = \begin{pmatrix} 1 & - & - & 1 \\ - & 1 & 1 & - \\ - & 1 & 1 & - \\ 1 & - & - & 1 \end{pmatrix}$$

Then $H = \mathrm{circ}(C_1, C_2, C_3, C_4)$ is the following matrix,

$$
\begin{bmatrix}
1&1&1&1&1&1&-&-&1&-&1&-&1&-&-&1\\
1&1&1&1&1&1&-&-&-&1&-&1&-&1&1&-\\
1&1&1&1&-&-&1&1&1&-&1&-&-&1&1&-\\
1&1&1&1&-&-&1&1&-&1&-&1&1&-&-&1\\
1&-&-&1&1&1&1&1&1&1&-&-&1&-&1&-\\
-&1&1&-&1&1&1&1&1&1&-&-&-&1&-&1\\
-&1&1&-&1&1&1&1&-&-&1&1&1&-&1&-\\
1&-&-&1&1&1&1&1&-&-&1&1&-&1&-&1\\
1&-&1&-&1&-&-&1&1&1&1&1&1&1&-&-\\
-&1&-&1&-&1&1&-&1&1&1&1&1&1&-&-\\
1&-&1&-&-&1&1&-&1&1&1&1&-&-&1&1\\
-&1&-&1&1&-&-&1&1&1&1&1&-&-&1&1\\
1&1&-&-&1&-&1&-&1&-&-&1&1&1&1&1\\
1&1&-&-&-&1&-&1&-&1&1&-&1&1&1&1\\
-&-&1&1&1&-&1&-&-&1&1&-&1&1&1&1\\
-&-&1&1&-&1&-&1&1&-&-&1&1&1&1&1
\end{bmatrix}
$$

Lemma 3. *Let H be a Bush-type Hadamard matrix of order $4n^2$. Let $H = H^+ - H^-$. Then each of H^+ and $H^- + I_{2n} \otimes J_{2n}$ is a symmetric $(4n^2, 2n^2 + n, n^2 + n)$–design. Furthermore, the two designs share the diagonal block entries $I_{2n} \otimes J_{2n}$.*

Proof. The proof is simple, but we present an unusual proof here which inspires a similar proof for Theorem 8; the main result of the paper.

First note that,

$$H^+ + H^- = J_{2n} \otimes J_{2n}.$$

So, $2H^+ = H + J_{2n} \otimes J_{2n}$. Now,

$$
\begin{aligned}
4H^+H^{+t} &= (H + J_{2n} \otimes J_{2n})(H^t + J_{2n} \otimes J_{2n}) \\
&= HH^t + 4n^2 J_{2n} \otimes J_{2n} + 4(n^2 + n)) J_{2n} \otimes J_{2n} \\
&= 4n^2 I_{4n^2} + 4(n^2 + n) J_{2n} \otimes J_{2n}.
\end{aligned}
$$

Therefore, $H^+H^{+t} = n^2 I_{4n^2} + (n^2 + n) J_{2n} \otimes J_{2n}$. This means that H^+ is a symmetric $(4n^2, 2n^2 + n, n^2 + n)$–design. It is similarly simple to see that $H^- + I_{2n} \otimes J_{2n}$ is also a symmetric $(4n^2, 2n^2 + n, n^2 + n)$–design. Note that Bush–typeness is essential for H to be a twin design. Computer computation shows that the two designs are different in general.

Example 4. Let H be the matrix of example 2, then, H^{+} is the following matrix,

$$\begin{bmatrix}
1&1&1&1&1&1&0&0&1&0&1&0&1&0&0&1\\
1&1&1&1&1&1&0&0&0&1&0&1&0&1&1&0\\
1&1&1&1&0&0&1&1&1&0&1&0&0&1&1&0\\
1&1&1&1&0&0&1&1&0&1&0&1&1&0&0&1\\
1&0&0&1&1&1&1&1&1&1&0&0&1&0&1&0\\
0&1&1&0&1&1&1&1&1&1&0&0&0&1&0&1\\
0&1&1&0&1&1&1&1&0&0&1&1&1&0&1&0\\
1&0&0&1&1&1&1&1&0&0&1&1&0&1&0&1\\
1&0&1&0&1&0&0&1&1&1&1&1&1&1&0&0\\
0&1&0&1&0&1&1&0&1&1&1&1&1&1&0&0\\
1&0&1&0&0&1&1&0&1&1&1&1&0&0&1&1\\
0&1&0&1&1&0&0&1&1&1&1&1&0&0&1&1\\
1&1&0&0&1&0&1&0&1&0&0&1&1&1&1&1\\
1&1&0&0&0&1&0&1&0&1&1&0&1&1&1&1\\
0&0&1&1&1&0&1&0&0&1&1&0&1&1&1&1\\
0&0&1&1&0&1&0&1&1&0&0&1&1&1&1&1
\end{bmatrix}$$

and the matrix $H^{-}+I_{2n}\otimes J_{2n}$ is

$$\begin{bmatrix}
1&1&1&1&0&0&1&1&0&1&0&1&0&1&1&0\\
1&1&1&1&0&0&1&1&1&0&1&0&1&0&0&1\\
1&1&1&1&1&1&0&0&0&1&0&1&1&0&0&1\\
1&1&1&1&1&1&0&0&1&0&1&0&0&1&1&0\\
0&1&1&0&1&1&1&1&0&0&1&1&0&-&0&1\\
1&0&0&1&1&1&1&1&0&0&1&1&1&0&1&0\\
1&0&0&1&1&1&1&1&1&1&0&0&0&1&0&1\\
0&1&1&0&1&1&1&1&1&1&0&0&1&0&1&0\\
0&1&0&1&0&1&1&0&1&1&1&1&0&0&1&1\\
1&0&1&0&1&0&0&1&1&1&1&1&0&0&1&1\\
0&1&0&1&1&0&0&1&1&1&1&1&1&1&0&0\\
1&0&1&0&0&1&1&1&1&1&1&1&1&1&0&0\\
0&0&1&1&0&1&0&1&0&1&1&0&1&1&1&1\\
0&0&1&1&1&0&1&0&1&0&0&1&1&1&1&1\\
1&1&0&0&0&1&0&1&1&0&0&1&1&1&1&1\\
1&1&0&0&1&0&1&0&0&1&1&0&1&1&1&1
\end{bmatrix}$$

Note that the two design share only the diagonal block entries.

3 A cyclic subgroup of signed permutation matrices

Let SP_m be the set of all signed permutation matrices of order m. Let $U=\text{circ}(0,1,0,\ldots,0)$ be the circulant shift permutation matrix of order $2n$ (this is a circulant matrix of order $2n$ with first row $010\ldots0$) and $N=\text{diag}(-,1,1,\ldots,1)$ be the diagonal matrix of order $2n$ with -1 at the $(1,1)$-position and 1 elsewhere on the diagonal. Let $E=UN$, then E is in SP_{2n}. Let $G_{4n}=\{\gamma^i=E^i\otimes I_{2n}: i=1,2,\ldots,4n\}=\prec\gamma\succ$.

Lemma 5. *G_{4n} is a cyclic subgroup of SP_{4n^2}, of order $4n$.*

Proof. For $1 \leq r \leq 2n$, $(UN)^r$ is U^r with its first r columns negated. Thus $\gamma^{2n} = E^{2n} \otimes I_{2n} = -I_{2n} \otimes I_{2n} = -I_{4n^2}$. It now follows that G_{4n} is a cyclic subgroup of SP_{4n^2}, of order $4n$.

Note that G_{4n} is a (signed) group subgroup of SP_{4n^2} and $\sum_{\gamma \in G_{4n}} \gamma = 0$.

Example 6. Let $n = 2$ in lemma 5, then

$$\gamma = E \otimes I_4 = \begin{pmatrix} 0 & I_4 & 0 & 0 \\ 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = E^2 \otimes I_4 = \begin{pmatrix} 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = E^3 \otimes I_4 = \begin{pmatrix} 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \\ 0 & 0 & -I_4 & 0 \end{pmatrix}$$

$$\gamma^4 = E^4 \otimes I_4 = \begin{pmatrix} -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \\ 0 & 0 & -I_4 & 0 \\ 0 & 0 & 0 & -I_4 \end{pmatrix}$$

$$\gamma^{4+j} = E^{(4+j)} \otimes I_4 = -E^j \otimes I_4 = -\gamma^j, \, j = 1, 2, 3, 4.$$

So for this example, $G_8 = \{\gamma^i = E^i \otimes I_4 \colon i = 1, 2, \ldots, 8\}$ is the cyclic subgroup of SP_{16}, of order 8.

Lemma 7. *Let $q = (2n+1)^2$ be a prime power. Then there is a balanced weighing matrix $BGW(q^m + q^{m-1} + \cdots + q + 1, q^m, q^m - q^{m-1})$ over the cyclic group G_{4n} for each positive integer m.*

Proof. Note that $4n$ is a divisor of $q - 1$ and apply [3], IV.4.22, or [4], Theorem 2.2.

4 The Siamese Twin Designs

Ionin [6] was the first to investigate symmetric designs with parameters $\nu = 4(q^m + q^{m-1} + \cdots + q + 1)n^2$, $\kappa = q^m(2n^2 + n)$, $\lambda = q^m(n^2 + n)$ for every positive integer m and $n = 3$. In [7], the author considered the case where $q = (2n-1)^2$ was a prime power. We now consider the case where $q = (2n+1)^2$ is a prime power.

The following is the main result of this paper.

Theorem 8. *Let H be a Bush–type Hadamard matrix of order $4n^2$ with $q=(2n+1)^2$ a prime power. Then there is a weighing matrix*

$$W(4(q^m+q^{m-1}+\cdots+q+1)n^2, 4q^mn^2)$$

which can be used to construct a Siamese twin design with parameters

$$\nu_m = 4(q^m+q^{m-1}+\cdots+q+1)n^2, \quad \kappa_m = q^m(2n^2+n), \quad \lambda_m = q^m(n^2+n),$$

for each positive integer m.

Proof. Let m be a positive integer. Let $W=[w_{ij}]$ be the balanced generalized weighing matrix $BGW(\nu,\kappa,\lambda)$ of Lemma 7, where $\nu = q^m+q^{m-1}+\cdots+q+1$, $\kappa=q^m$, $\lambda = q^m-q^{m-1}$. Consider the block matrix $A=[Hw_{ij}]$ of order $4\nu n^2$. Let $AA^t=[B_{kl}]$. For $k\neq l$,

$$\begin{aligned} B_{kl} &= \sum_{j=1}^{\nu} Hw_{kj}(Hw_{jl})^t \\ &= \sum_{j=1}^{\nu} H(w_{kj}{w_{lj}}^t)H^t \\ &= H(\sum_{j=1}^{\nu} w_{kj}{w_{lj}}^t)\,H^t \\ &= H(\sum_{\gamma\in G_{4n}} \frac{\lambda}{4n}\gamma)\,H^t = 0. \end{aligned}$$

For $k=l$,

$$\begin{aligned} B_{kl} &= H\sum_{j=1}^{\nu}(w_{kj}{w_{lj}}^t)\,H^t \\ &= \kappa HH^t \\ &= 4n^2\kappa I_{2n}. \end{aligned}$$

So A is a weighing matrix $W(4n^2\nu, 4n^2\kappa)$. Now, let

$$D = [Mw_{ij} + (I_{2n}\otimes J_{2n})|w_{ij}|],$$

where $M = H - I_{2n}\otimes J_{2n}$. This matrix (which is obtained from A by negating some entries) contains a symmetric designs with the parameters, $\nu_m = 4(q^m+q^{m-1}+\cdots+q+1)n^2$, $\kappa_m = q^m(2n^2+n)$, $\lambda_m = q^m(n^2+n)$ for every positive integer m. To see this, let

$$D = D^+ - D^- = [Mw_{ij} + (I_{2n}\otimes J_{2n})|w_{ij}|].$$

Now observe that

$$D^+ + D^- = [\big(J_{2n}\otimes J_{2n}\big)|w_{ij}|].$$

Therefore,

$$2D^{+} = [Mw_{ij} + \big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|].$$

First note that for all i,j,k,l,

$$\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|)(Mw_{kl})^{t} = (Mw_{kl})\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|)^{t} = 0.$$

Also note that a calculation on $[Mw_{ij}]$ akin of that on A above would give,

$$[Mw_{ij}][Mw_{ij}]^{t} = \kappa(I_{\nu}\otimes MM^{t}).$$

So, we have,

$$\begin{aligned}4D^{+}D^{+t} &= [\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|][\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|]^{t} + [Mw_{ij}][Mw_{ij}]^{t}\\ &= [\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|][\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{ij}|]^{t} + \kappa(I_{\nu}\otimes MM^{t}).\end{aligned}$$

For $k\neq l$,

$$\begin{aligned}&\sum_{j=1}^{\nu}\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{kj}|(\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{jl}|)^{t}\\ &= \big((J_{2n}+I_{2n})\otimes J_{2n}\big)(\sum_{j=1}^{\nu}(|w_{kj}||w_{lj}{}^{t}|)\big((J_{2n}+I_{2n})\otimes J_{2n}\big)\\ &= \frac{\lambda}{2n}\big((J_{2n}+I_{2n})\otimes J_{2n}\big)(J_{2n}\otimes I_{2n})\big((J_{2n}+I_{2n})\otimes J_{2n}\big)\\ &= \frac{\lambda}{2n}\big((J_{2n}+I_{2n})\otimes J_{2n}\big)\big(J_{2n}\otimes I_{2n}\big)\big((J_{2n}+I_{2n})\otimes J_{2n}\big)\\ &= \frac{\lambda}{2n}\big(2n+1)^{2}J_{2n}\otimes 2nJ_{2n}\\ &= \lambda(2n+1)^{2}J_{2n}\otimes J_{2n}\\ &= 4n(n+1)q^{m}J_{2n}\otimes J_{2n}.\end{aligned}$$

Therefore, all the (k,l), $1\leq k\neq l\leq\nu$ blocks of the matrix $D^{+}D^{+t}$ consist of the matrix $n(n+1)q^{m}J_{2n}\otimes J_{2n}$.
For $k=l$ we have,

$$\begin{aligned}&\sum_{j=1}^{\nu}\big(\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{kj}|\big)\big(\big((J_{2n}+I_{2n})\otimes J_{2n}\big)|w_{kj}|\big)^{t}+\\ &\sum_{j=1}^{\nu}Mw_{kj}(Mw_{kj})^{t}\\ &= \kappa\big((J_{2n}+I_{2n})\otimes J_{2n}\big)\big((J_{2n}+I_{2n})\otimes J_{2n}\big)^{t} + MM^{t})\\ &= q^{m}\big(4n^{2}I_{4n^{2}} + 4n(n+1)J_{2n}\otimes J_{2n}\big).\end{aligned}$$

From this we conclude that all the $(k,k), 1\leq k\leq\nu$ blocks of the matrix $D^{+}D^{+t}$ consist of the matrix $q^{m}\big(n^{2}I_{4n^{2}} + n(n+1)J_{2n}\otimes J_{2n}\big)$. Therefore, D^{+} is a symmetric $(\nu_{m},\kappa_{m},\lambda_{m})$–design.

By a similar argument we can show that

$$\frac{1}{2}[-Mw_{ij} + \big((J_{2n} + I_{2n}) \otimes J_{2n}\big)|w_{ij}|]$$

is also a symmetric $(\nu_m, \kappa_m, \lambda_m)$–design. Please note that the two designs share the part $(I_{2n} \otimes J_{2n})|w_{ij}|$ and both could be obtained from the weighing matrix A.

Corollary 9. *Let $4n$ be the order of a Hadamard matrix with $q = (4n+1)^2$ a prime power. Then there is a weighing matrix*

$$W(16(q^m + q^{m-1} + \cdots + q + 1)n^2, 16q^m n^2)$$

from which two symmetric designs are generated with the parameters

$$\nu_m = 16(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa_m = q^m(8n^2 + 2n), \quad \lambda_m = q^m(4n^2 + 2n)$$

for every positive integer m.

Proof. This follows from Theorems 1 and 8.

Remark 10. Corollary 9 provides a new class of symmetric designs.

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References

1. K.A. Bush: Unbalanced Hadamard matrices and finite projective planes of even order, JCT, 11(1971), pp. 38–44.
2. Frans C. Bussemaker, Willem Haemers and Edward Spence: The search for pseudo orthogonal Latin squares of order six, preprint.
3. Warwick de Launey: Section on “Bhaskar Rao designs,” CRC Handbook of Combinatorial Designs, edited by Charles J. Colbourn and Jeffrey H. Dinitz, Kluwer Academic Press, 1996.
4. Dieter Jungnickel and Vladimir Tonchev: Perfect Codes and Balanced Weighing Matrices, To appear (1999).
5. Zvonimir Janko, Hadi Kharaghani and Vladimir D. Tonchev, The existence of a Bush–type Hadamard matrix of order 324 and two new infinite classes of symmetric designs, preprint.
6. Y.J. Ionin: New symmetric designs from regular Hadamard matrices, The Electronic Journal of Combinatorics, Vol. 5, No.1 (1998), R1.
7. H. Kharaghani: On the twin designs with the Ionin–type parameters, The Electronic Journal of Combinatorics, (2000), #R1.
8. H. Kharaghani: New classes of weighing matrices, ARS comb., 19(1985), pp. 69–72.