Program
Alberta-Montana Combinatorics and Algorithms Days (2023)
Banff International Research Station, Banff, Alberta

Friday June 23	**Speaker or Event**	**Title**
by evening	*Check-in & Registration at the front desk in the Professional Development Centre*	

Saturday June 24	**Speaker or Event**	**Title**
8:20-8:25	*Welcome address by BIRS staff*	
8:30-9:00	Davoud Abdi, U. of Calgary	Counterexamples to Conjectures of Thommassé and Bonato-Tardif, what is next?
9:00-9:30	Martin Müller, U. of Alberta	Towards Efficient First-Principles Algorithms for Sums of Combinatorial Games
9:30-10:00	Xinyue Chen, U. of Alberta — presented by Ryan Hayward	Exploring Hex Solving with the Benze Hex Solver and DFPN Algorithm
10:00-10:30	*Coffee break*	
10:30-11:30	Svenja Huntemann, Concordia U. of Edmonton	Values, Temperatures, and Enumeration of Placement Games
11:30-12:00	Mahya Jamshidian, U. of Alberta	On improving approximation factors for Minimum Sum of Radii, Diameters, and Squared Radii
12:00-1:00pm	*Lunch*	
1:00-1:20	*Group Photo in TCPL Foyer near the talk venue (or outside weather permitting)*	
1:30-2:30	Stephanie van Willigenburg, U. B.C.	The $(3+1)$-free conjecture of chromatic symmetric functions
2:30-3:00	Zac Friggstad, U. of Alberta	Approximating Minimum-Sum Colourings of Chordal Graphs
3:00-3:30	*Coffee Break*	
3:30-4:00	Caleb Van't Land, U. of Lethbridge	A Maximal Set of Unbiased Butson Hadamard Matrices
4:00-5:00	Dave Witte Morris, U. of Lethbridge	On vertex-transitive graphs with a unique hamiltonian circle
5:00-5:30	Van Magnan, U. of Montana	Flower Power
5:30-6:00	Amarpreet Rattan, Simon Fraser U.	Parking functions, trees and factorizations in the symmetric group
6:00-7:30	*Dinner*	

Sunday June 25	**Speaker or Event**	**Title**
8:30-9:30	Cory Palmer, U. of Montana	A survey of extremal co-degree problems for hypergraphs
9:30-10:00	Anna Halfpap, U. of Montana	General Properties of the Positive Co-degree Turán Number
10:00-10:30	*Coffee break (note: room check out is by 11:00)*	
10:30-11:30	Rei Safavi-Naini, U. of Calgary	Key agreement for long term secure communication
11:30-12:00	Daniel P. Johnston, Trinity College, Hartford, CT	Rainbow Saturation
12:00pm	*Lunch*	

ABSTRACTS

Alberta-Montana Combinatorics and Algorithms Days
Banff International Research Station
Banff, Alberta
Friday June 23 – Sunday June 25, 2023

Invited Speakers (in alphabetical order by speaker's name)

Values, Temperatures, and Enumeration of Placement Games

Svenja Huntemann
Department of Mathematical and Physical Sciences
Concordia University of Edmonton

Placement games are two-player games played on finite graphs in which the players take turns placing tokens, without moving or removing them later. Many of these games have been studied extensively, but few are completely solved. The value of a game indicates how large the advantage of the winning player is, while temperature gives an indication of the urgency to move first. We consider the values and temperatures of several placement games.

Enumeration of positions in placement games is a relatively new topic. The number of positions of a certain type in relation to all positions gives an indication of the complexity of analysis and which analysis tool might be most efficient. We will discuss the enumeration of positions in two different types of placement games.

This is joint work with Neil McKay, Lexi Nash, and Craig Tennenhouse.

A survey of extremal co-degree problems for hypergraphs

Cory Palmer
Department of Mathematical Sciences
University of Montana

An *r-graph* is a hypergraph whose edges (called *r-edges*) all have size r, i.e., an r-uniform hypergraph. A hypergraph is F*-free* if it does not contain F as a subhypergraph. The *Turán number* $\mathrm{ex}(n,F)$ is the maximum number of edges in an n-vertex F-free r-graph. For graphs (i.e., when $r=2$), much is known about the Turán number, but when $r \geq 3$, very few exact results are known. Notoriously, the Turán number of the complete 4-vertex 3-graph is unknown.

Instead of maximizing the number of edges, we may examine degree versions of the Turán number. For $r=2$, this is equivalent to determining the function $\mathrm{ex}(n,F)$, but when $r \geq 3$ there are multiple interpretations of "degree." In an r-graph, the *co-degree* of an $(r-1)$-set S is the number of edges containing S. The minimum co-degree over all $(r-1)$-sets in H is denoted $\delta_{r-1}(H)$. The *minimum positive co-degree* is denoted $\delta^{+}_{r-1}(H)$. The *co-degree Turán number* $\mathrm{coex}(n,F)$ is the maximum value of $\delta_{r-1}(H)$ over all n-vertex F-free r-graphs. While the *positive co-degree Turán number* $\mathrm{co}^{+}\mathrm{ex}(n,F)$ is the corresponding measure for positive co-degree.

In this talk we will give a history of these three parameters in hypergraphs and then focus on recent developments on the positive co-degree problem.

Key agreement for long term secure communication

Rei Safavi-Naini
Department of Mathematics and Statistics
University of Calgary

Key agreement protocols allow Alice and Bob to share a secret key by communicating over an insecure channel. They are a fundamental cryptographic primitive that are used in every secure communication over the Internet.

In this talk we look at approaches to this problem and in particular when the adversary has unlimited computation.

The $(3+1)$-free conjecture of chromatic symmetric functions

Stephanie van Willigenburg
Department of Mathematics
University of British Columbia

The chromatic symmetric function, dating from 1995, is a generalization of the chromatic polynomial. A famed conjecture on it, called the Stanley-Stembridge $(3+1)$-free conjecture, has been the focus of much research lately. In this talk we will be introduced to the chromatic symmetric function, the $(3+1)$-free conjecture, new cases and tools for resolving it, and answer another question of Stanley of whether the $(3+1)$-free conjecture can be widened. This talk requires no prior knowledge.

On vertex-transitive graphs with a unique hamiltonian circle

Dave Witte Morris
Department of Mathematics and Computer Science
University of Lethbridge

It is conjectured that cycles are the only finite regular graphs that have a unique hamiltonian cycle. Mateja Šajna and Andrew Wagner proved in 2014 that the conjecture is true in the special case where the graph is vertex-transitive, which means there is an automorphism that takes any vertex to any other vertex.

We will discuss the generalization of the vertex-transitive case to infinite graphs. (Infinite graphs do not have "hamiltonian cycles," but there are natural analogues.) It is not difficult to solve the case where the graph has only finitely many ends, but the case where there are infinitely many ends is not yet understood. Sean Legge recently constructed the first family of infinite examples. Joint work with Bobby Miraftab (and with help from Agelos Georgakopoulos) constructed examples that are Cayley graphs on free groups. All of these examples are outerplanar, but we do not know whether that is always true.

Contributed Talks (in alphabetical order by speaker's name)

Counterexamples to Conjectures of Thommassé and Bonato-Tardif, what is next?

Davoud Abdi

Department of Mathematics and Statistics
University of Calgary

Two structures are called *equimorphic* when each embeds in the other; we may also say that one is a *sibling* of the other. Equimorphic finite structures are necessarily isomorphic, but this is no longer the case for infinite structures. For instance, the rational numbers, considered as a linear order, has continuum many siblings, up to isomorphism. Thomassé (2000) conjectured that a countable relation has either one, countably or continuum many siblings, up to isomorphism. There is a special case of interest stating that a relational structure of any cardinality has one or infinitely many siblings. This is connected to a conjecture of Bonato-Tardif stating that a tree has one or infinitely many siblings.

This talk will mention those structures for which the conjectures have been verified by giving historical progress. Then, the counterexamples to the conjectures will be presented. Finally, we generalise the sibling notion from embedding to quasi-orders and state open problems related.

Exploring Hex Solving with the Benze Hex Solver and DFPN Algorithm

Xinyue Chen
Department of Computing Science
University of Alberta

Hex is a two-player zero-sum game that was invented in the 1940s. For an empty n x n board, using proof by contradiction, the first player will always win under perfect play. We are interested in determining which positions can lead the first player to victory and which cannot. Currently, only a few opening moves have been solved for a 10 x 10 board.

We are currently using the Benze Hex Solver with the DFPN algorithm to solve the Hex game. In this talk, we will discuss a few strategies we are working on that may make the solving algorithm more efficient. We are working on guiding the solver to solve difficult paths (determined by the proof number and disproof number) first. Additionally, we are using downsize game strategies and decompositions to improve move ordering.

Approximating Minimum-Sum Colourings of Chordal Graphs

Zac Friggstad
Department of Computing Science
University of Alberta

In the *minimum sum colouring* problem (**MSC**) we colour a graph with positive integers with the goal of minimizing the average colour given to a node. Halldórson, Kortsarz, and Shachnai (2003) presented an algorithm that approximates **MSC** within a factor of 1.796 in interval graphs and, more generally, in any graph class where the largest k-colourable induced subgraph problem can be solved in polynomial time.

We study **MSC** in chordal graphs, i.e. graphs that do not include an induced cycle of length ≥ 4 (a generalization of interval graphs). While the largest k-colourable induced subgraph problem is **NP**-hard in chordal graphs, we show how to approximate it within any desired constant factor in polynomial time. Combining this with relatively new linear-programming based techniques for minimum latency problems, we obtain a matching 1.796-approximation for **MSC** in chordal graphs.

This is joint work with Ian DeHaan.

General Properties of the Positive Co-degree Turán Number

Anna Halfpap
Department of Mathematical Sciences
University of Montana

In an r-graph, the co-degree of a set S of $(r-1)$ vertices is simply the number of edges which contain S. The *minimum positive co-degree* of an r-graph H is the largest integer k such that, if an $(r-1)$-set of vertices S is contained in at least one r-edge, then S is contained in at least k r-edges. For a fixed r-graph F, the positive co-degree Turán number of F is the largest possible minimum positive co-degree in an n-vertex r-graph which does not contain F as a subhypergraph.

In addition to computing the positive co-degree Turán number of specific r-graphs, there are many natural questions regarding the general behavior of the positive co-degree Turán function. Other Turán-type functions are known to be "well-behaved" in certain ways; for example, their scaled density limits always exist. In this talk, we shall discuss some general properties of well-studied extremal functions, and sketch proofs of analogous properties for the positive co-degree Turán function.

On improving approximation factors for Minimum Sum of Radii, Diameters, and Squared Radii

Mahya Jamshidian
Department of Computing Science
University of Alberta

In this talk, we present an improved approximation algorithm for three related problems. In the Minimum Sum of Radii clustering problem (MSR), we aim to select k balls in a metric space to cover all points while minimizing the sum of the radii. In the related Minimum Sum of Diameters clustering problem (MSD), we are to pick k clusters to cover all the points such that the sum of diameters of all the clusters is minimized. At last, in the Minimum Sum of Squared Radii problem (MSSR), the goal is to choose k balls, similar to MSR. However in MSSR, the goal is to minimize the sum of squares of radii of the balls.

We present a 3.389-approximation for MSR and a 6.546-approximation for MSD, improving over respective 3.504 and 7.008 developed by Charkar and Panigrahy (2001).

In particular, our guarantee for MSD is better than twice our guarantee for MSR. In the case of MSSR, the best known approximation guarantee is $4.(540)^2$ based on the work of Bhowmick, Inamdar, and Varadarajan in their general analysis of the t-Metric Multicover Problem. Our work describes an 11.078-approximation algorithm for MSSR with a similar approach to the MSR and MSD cases.

Rainbow Saturation

Daniel P. Johnston
Department of Mathematics
Trinity College - Hartford, CT

A graph G is rainbow H-saturated if there is some proper edge coloring of G which is rainbow H-free (that is, it has no copy of H whose edges are all colored distinctly), but where the addition of any edge makes such a rainbow H-free coloring impossible. Taking the maximum number of edges in a rainbow H-saturated graph recovers the rainbow Turán numbers whose systematic study was begun by Keevash, Mubayi, Sudakov, and Verstraëte. In this talk, we introduce and examine the corresponding *rainbow saturation number* – the minimum number of edges among all rainbow H-free graphs.

This is joint work with Neal Bushaw (VCU) and Puck Rombach (UVM).

Flower Power

Van Magnan
Department of Mathematical Sciences
University of Montana

In extremal set theory, we often ask questions about maximizing the size of a family subject to certain intersecting conditions. The Erdős-Ko-Rado Theorem describes how the maximum size of any intersecting family is achieved by a 'trivially' intersecting family, in which all members contain a common element. Several generalizations of this problem exist, including the following:

Maximize the quantity $|\mathcal{F}| - \Delta(\mathcal{F})$, where $\Delta(\mathcal{F})$ is the max degree of an element in the family, such that the family remains intersecting.

The above problem maximizes the *diversity* of the family. We define the family's flower base, which combines the methods of delta system and transversal analysis, and show how it can be used to show results in the field. We then partially answer a generalization of the above question, finding extremal constructions maximizing $|\mathcal{F}| - C \cdot \Delta(\mathcal{F})$ for any constant $C \in [0, 7/3)$.

Towards Efficient First-Principles Algorithms for Sums of Combinatorial Games

Martin Müller

Department of Computing Science
University of Alberta

In combinatorial games, the most fundamental question is "who wins?" Algorithms for solving combinatorial games try to answer this question efficiently. One main tool is exploiting sum game structure, in the case when a game is a sum of independent subgames.

Software such as Siegel's CGSuite is built around the fundamental concept of canonical form, as defined in "On Numbers and Games" and "Winning Ways". However, for solving sums of even moderately complex subgames, the cost of computing canonical forms can become a computational bottleneck. In this talk we report on ongoing work to solve sum games by search, directly exploiting first-principles properties of the subgames. We focus on answering only boolean questions of the form "does the first player win or lose", and develop algorithms that are streamlined for this purpose and avoid all canonical form computations.

After some motivational examples, we define our basic framework for search and show first computational results on $1 \times n$ boards for combinatorial games such as Clobber and NoGo.

A Maximal Set of Unbiased Butson Hadamard Matrices

Caleb Van't Land

Department of Mathematics and Computer Science
University of Lethbridge

A square matrix of order n, say H, whose entries are drawn from the m-th complex roots of unity and which satisfies $HH^* = nI$ is called a Butson Hadamard matrix and is denoted as BH(n, m).

A Bush-type Butson Hadamard matrix is a Butson Hadamard matrix of order n^2 which is subdivided into n^2 blocks of order n such that blocks on the main diagonal consist entirely of 1s, and all other blocks have row and column sums of 0.

A pair of BH(n^2, m)'s, say H_1 and H_2, are said to be unbiased if $n^{-1}H_1H_2^*$ is also a BH(n^2, m). A collection of BH(n^2, m)'s is mutually unbiased if each pair is unbiased.

For a prime power q, a set of q mutually unbiased BH(q^2, q)'s will be constructed and shown to be **maximal**.

This is joint work with H. Kharaghani, T. Pender, and V. Zaitsev.