ABSTRACTS

Western Canada Linear Algebra Meeting (W-CLAM) 2012 University of Lethbridge Lethbridge, Alberta May 12 – 13, 2012

Invited Speakers

Randomized Spectral Divide-and-Conquer: a Communication-Optimal Approach to the Non-symmetric Eigenproblem

Ioana Dumitriu University of Washington, Seattle, Washington

Algorithms have two costs: arithmetic (number of operations) and communication (number of messages and words being moved between levels of a memory hierarchy, or between processors over a network). For large problems, the cost of communication can greatly exceed the cost of arithmetic, hence the need for communication-avoiding or communication-minimizing algorithms. In 2009, Ballard, Demmel, Holtz, and Schwartz have given lower bounds on the communication costs for the eigenproblem. Subsequently, we have found parallel and sequential algorithms that attain these lower bounds using randomization. (To date, we know of no communication-optimal algorithms for the non-symmetric eigenproblem that are deterministic.)

I will present these algorithms for the case of the non-symmetric eigenproblem, explain why they are numerically stable, and how randomization "smooths out" potential kinks.

This is joint work with Grey Ballard and James Demmel.

Quantum operations and completely positive linear maps

Chi-Kwong Li

College of William and Mary, University of Hong Kong

In quantum information science, it is important to know whether a given set of quantum states can be transformed to another set of quantum states by quantum operations In mathematical terms, one would like to determine the necessary and sufficient condition for the existence of a trace preserving completely positive map which will send a given set of density matrices to another set of density matrices. In this talk, recent results and problems on this question will be discussed.

Local-to-Global Properties of Semigroups of Matrices Heydar Radjavi University of Waterloo, Waterloo, ON

Here is the general theme of this talk: knowing that something about a group or semigroup G of matrices is "small" in some sense, we want to conclude if possible that G itself is small in the same sense. The classical example of this is the theorem that an irreducible group of complex matrices is finite if traces of its elements form a finite set. We can also replace "finite" with "bounded". Several other measures of smallness will be considered; for example, representability of G in a smaller field than the one we started with.

Common invariant subspace and commuting matrices Gérald Bourgeois

GAATI, Université de la polynésie française, FAA'A, Tahiti, Polynésie Française

Let *K* be a perfect field. Let $A, B \in \mathcal{M}_n(K)$ be such that they have a common invariant proper vector subspace of dimension *k* over an extension field of *K* and the characteristic polynomial of *A* is irreducible over *K*. If $k \in \{1, n - 1\}$ then *A* and *B* commute. If the Galois group *G* of the characteristic polynomial of *A* is S_n , then, for every *k*, *A* and *B* commute. If $k \in \{2, n - 2\}$ and $G = A_n$ then AB = BA. This conclusion may be false if *G* is not S_n or A_n .

Matrix patterns that allow arbitrary inertia

Mike Cavers

Dept. Mathematics & Statistics, University of Calgary, Calgary, Alberta

In this talk, I will discuss techniques that may be used to show a matrix pattern allows arbitrary inertia. In particular, a modification of the Nilpotent-Jacobian method is presented along with examples to demonstrate how it can be used to prove a sign pattern allows arbitrary inertia.

This is joint work with Shaun Fallat.

Inverse Problems for Hermitian Quadratic Matrix Polynomials

C. Chorianopoulos^{*} and P. Lancaster

Dept. of Mathematics and Statistics, University of Calgary, Calgary, Alberta

Given a nonsingular leading coefficient and a left divisor, we construct compatible right divisors in order to obtain an Hermitian quadratic matrix polynomial. Special emphasis is placed on the case of real symmetric systems. Further spectral properties are discussed.

Joint Spectral Radius approximation

Antonio Cicone

Dept. of Mathematics, Michigan State University, East Lansing, MI

Let \mathcal{F} be a finite set of matrices in $\mathbb{R}^{n \times n}$, it is possible to define $\rho(\mathcal{F})$ as the *joint spectral radius* or simply *JSR* of the set which is a generalization of the well known spectral radius of a matrix [G. C. Rota, G. Strang, A note on the joint spectral radius, *Indag. Math.*, vol. 22, 1960, pp 379–381]. The JSR evaluation proves to be useful in many contexts like in the construction of wavelets of compact support, in analyzing asymptotic behaviors of linear difference equations solutions with variable coefficients and many others. However this quantity proves to be hard to compute in general. Gripenberg in [G. Gripenberg, Computing the joint spectral radius, 1996] proposed an algorithm for computing lower bounds and upper bounds to $\rho(\mathcal{F})$ making use of a four member inequality and a branch & bound technique, while Blondel et al. developed a conic programming approach that proves to be helpful in the quest for a tighter upper bound [V. D. Blondel, R. M. Jungers, V. Y. Protasov, Joint spectral characteristics of matrices: a conic programming approach, 2010].

In this talk we describe a new method to compute the JSR that, following the ideas of Gripenberg and Blodel at al., makes use of semidefinite lifting procedures, ellipsoidal norms, conic programming methods, bisection and branch & bound techniques to achieve in a finite number of steps and up to a desired arbitrarily high precision lower bounds and upper bounds to $\rho(\mathfrak{F})$. We show the performance of this new algorithm compared with Gripenberg's one.

This contribution is a joint work with V.Y.Protasov (Moscow State University, Russia).

Title TBA

R. Craigen

Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba

Abstract TBA

Arveson's criterion for unitary similarity Douglas Farenick

Dept. of Mathematics & Statistics, University of Regina, Regina, Saskatchewan

Forty years ago in the Bulletin of the AMS, W.B. Arveson announced an important theorem concerning the unitary similarity problem for complex matrices, the proof of which appeared two years later in the second of a pair of long and influential papers in operator algebras. The importance of the operator algebraic content in these two papers has somewhat overshadowed Arveson's novel contribution to linear algebra. Therefore, in this lecture I will present Arveson's theorem on the unitary similarity problem and give the steps in a new and selfcontained proof of the result.

The Nilpotent-Centralizer Method

Colin M. Garnett University of Victoria, Victoria, B.C.

In this talk I will discuss the main topic from my dissertation, the Nilpotent-Centralizer method. This is a new method for recognizing spectrally arbitrary sign patterns. I will give a short description of the Nilpotent-Centralizer method and give some examples of its applications.

An analogue of Kreĭn's inequality with applications

Minghua Lin

Dept of Applied Math, University of Waterloo, Waterloo, Ontario

There are two notions of angle between vectors in \mathbb{C}^n that are frequently used in the literature. One is defined by means of the standard formula

$$\cos \Phi_{xy} = \frac{Re\langle x, y \rangle}{\|x\| \|y\|}, \quad x, y \in \mathbb{C}^n \setminus \{0\};$$

the other one is defined by

$$\cos \Psi_{xy} = rac{|\langle x, y
angle|}{||x||||y||}, \quad x, y \in \mathbb{C}^n \setminus \{0\}.$$

M.G. Kreĭn in 1969 discovered such an inequality:

 $\Phi_{xz} \leq \Phi_{xy} + \Phi_{yz}.$

In this talk, an analogue of Kreĭn's inequality is given, i.e.,

$$\Psi_{xz} \leq \Psi_{xy} + \Psi_{yz}.$$

Some interesting applications as well as the extension to canonical angles in subspaces are also presented.

Ratio of Matrices

Shahla Nasserasr

Dept. of Mathematics and Statistics, University of Regina, Regina, Saskatchewan

The question of whether or not a ratio of matrices (AB^{-1}) in a positivity class (positive definite, totally positive, invertible M-matrices, etc.) has a nested sequence of positive principal minors is considered. The classes for which this problem has been solved will be presented.

Spectral Set theory and the spectrum of Banach algebra elements

Rajesh Pereira Dept. of Mathematics and Statistics, University of Guelph, Guelph ON

In 1955, Helmut Wielandt gave regions in the complex plane which contain all the eigenvalues of the sum of two normal matrices. We generalize Wielandt's result both to non-normal matrices and to Banach algebras using von Neumann's theory of spectral sets. We examine some ways of generalizing the concept of normality to Banach algebras. We also give multiplicative versions of Wielandt's theorem.

This is joint work with Stephen Rush.

Some Inequalities of Majorization Type

Fuzhen Zhang Nova Southeastern University, Ft Lauderdale, Florida

This talk is concerned with matrix inequalities of majorization type. We show some basic majorization inequalities of vectors then apply them to derive matrix inequalities.

Biosynthesis of Cutin and Suberin From Free Monomers: Modeling of Control Processes

Peter Zizler Mount Royal University, Calgary, Alberta

Plants synthesize suberin to create a hydrophobic barrier to water or other environmental stresses. During this synthesis free monomers pass through the cell membrane and they go on to form polymers suberin (or cutin) on the cell wall depending on the type of plant cell material (ex. leaves or roots). As far as we know it is not clear whether this polymer formation process is protein controlled or whether its formation is a spontaneous random process. In our talk we present some probabilistic models that could provide insight to this question.

Inverses of symmetric, diagonally dominant positive matrices and applications

Christopher J. Hillar, Shaowei Lin, Andre Wibisono University of California, Berkeley

We prove tight bounds for the ∞ -norm of the inverse of a symmetric, diagonally dominant positive matrix J; in particular, we show that $||J^{-1}||_{\infty}$ is uniquely maximized among all such J. We also prove a new lower-bound form of Hadamard's inequality for the determinant of diagonally dominant positive matrices and an improved upper bound for diagonally balanced positive matrices. Applications of our results include numerical stability for linear systems, bounds on inverses of differentiable functions, and consistency of the maximum likelihood equations for random graph distributions.

Amy Streifel

Dept. of Mathematics, Washington State University, Pullman, WA

A skew-adjacency matrix *A* of a graph *G* is a $\{0, 1, -1\}$ -matrix, where $a_{ij} = -a_{ji} \in \{1, -1\}$ if $\{i, j\}$ is an edge in *G*, and $a_{ij} = a_{ji} = 0$ otherwise. There may be several non-equivalent skew-adjacency matrices for a particular graph. The skew-spectra of a graph is the set of all possible spectra realized by the skew-adjacency matrices associated with the graph. This poster will discuss the number of possible spectra associated with certain classes of graphs.