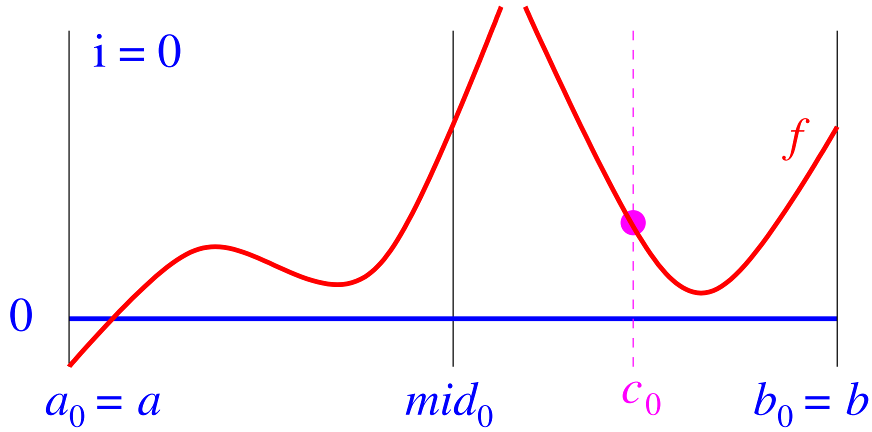
A proof that continuous implies bounded:

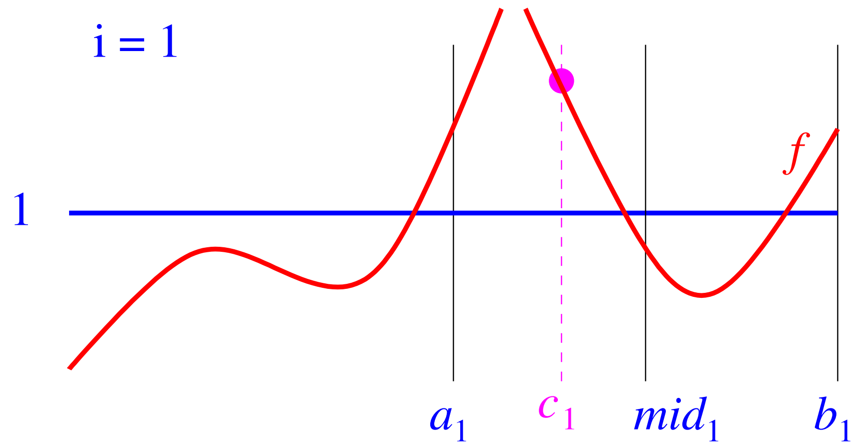
For continuous f on [a,b], show that for some M,

f(x) <= M for all x.

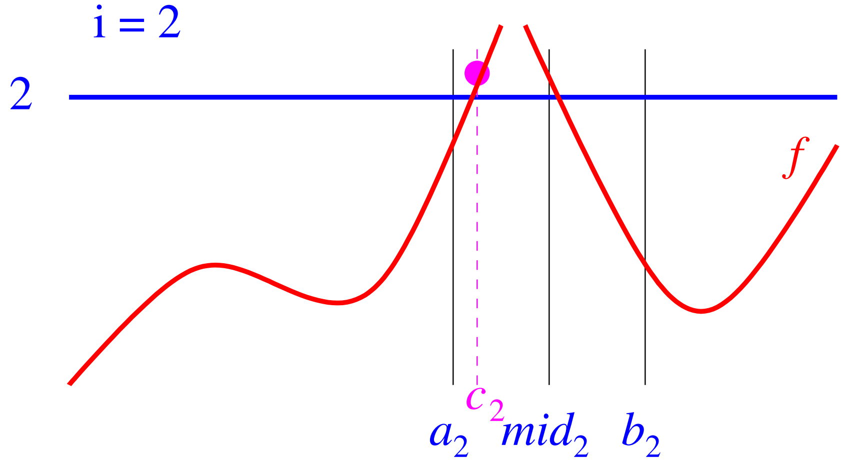


If not, then "for some M, f(x) <= M" is violated on at least one of the half intervals determined by mid_i.

If violated on the right half interval (as in this figure), then make mid_i the new left endpoint and in the right half interval select c_i with $f(c_i) >= i$ (i = 0, 1, 2, ...).



Similarly if on the left (as shown in this figure for next stage).



Repeat ad infinitum.

The left endpoints (which increase) and the right endpoints (which decrease) must limit to a number x since the distance apart is halved at each stage.
The c_i also limit to x and $f(c_i) \to f(x)$ since f is continuous.
But f(x) is a fixed number, whereas $f(c_i)$'s become arbitrarily large.

Note: The proof of "for some M, M <= f(x)" follows by applying above to −f.