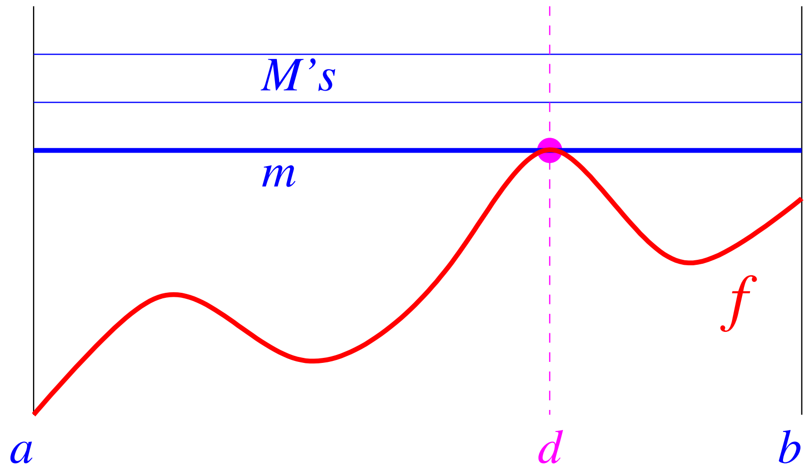
Proof of the Extreme Value Theorem:

For continuous f on [a,b], show that for some c and d,

f(c) <= f(x) <= f(d) for all x in [a,b].

Only prove f(x) <= f(d), since then the f(c) <= f(x) case follows by considering –f(x). We could give a proof similar to that for boundedness, but instead use that result.

By boundedness, f(x) <= M for some M.
Among such M's there is a minimum value m which has f(x) <= m.



Show f(d) = m for some d.
If not, then 1 / (m – f(x)) is defined everywhere on [a,b].
It is continuous, so by the "countinuous is bounded theorem", it is bounded:
For some k > 0, 1/ (m – f(x)) <= k.

Then 1/k <= m – f(x).

So f(x) <= m – 1/k, but this says m – 1/k would be a lower bound than m.