Gerschgorin's Circle Theorem

The eigenvalues of A lie in the union of circles center a_{ii} radius $\sum_{j\neq i} |a_{ij}|$.

Proof: The i-th component equation of $\lambda\mathbf{x} = A\mathbf{x}$ rewrites as $(\lambda - a_{ii})x_i = \sum_{j\neq i} a_{ij}x_j$. For non-zero x_i,

$$|\lambda - a_{ii}| \leq \sum_{j\neq i} |a_{ij}||x_j|/|x_i|.$$

Take i so that $|x_i|$ is maximal; note that not all x_i of an eigenvector can be zero. Then λ lies in the circle with indicated radius centered at a_{ii}.