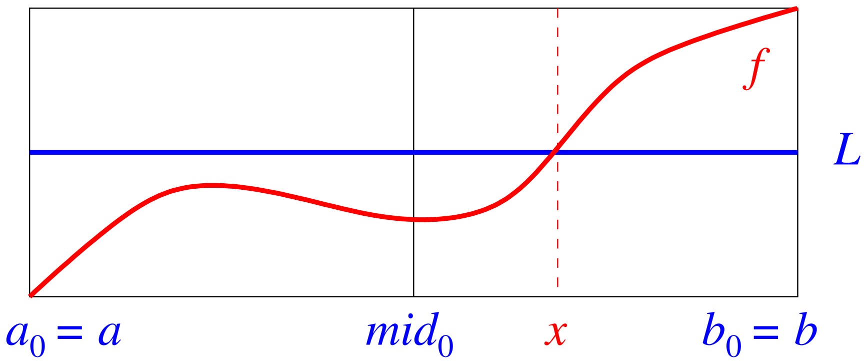
Proof of the Intermediate Value Theorem

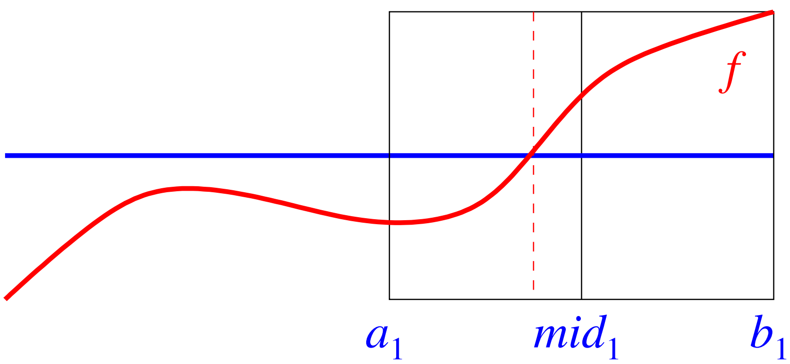
For continuous f on [a,b], show that

if $f(a) < L < f(b)$ then $f(x) = L$ for some $a < x < b$.

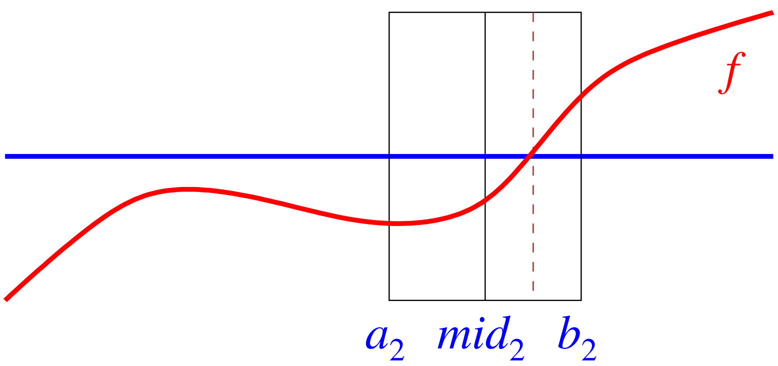
Note that the proof gives a method for finding x.
For case of L = 0, it finds a zero of f, one binary digit at a time.



Consider midpoint (mid).
If f(mid) = L then done.
Otherwise, as f(mid) < L or > L make mid the new left or right endpoint.



Repeat ad infinitum.



The left endpoints (which increase) and the right endpoints (which decrease) must limit to a number x since the distance apart is halved at each stage.
On each left endpoint a, $f(a) < L$ so since f is continuous $f(x) <= L$.
On each right endpoint b, $f(b) > L$ so since f is continuous $f(x) >= L$.
Thus $f(x) = L$.