ABSTRACTS

Second Prairie Discrete Mathematics Workshop
University of Lethbridge
Lethbridge, Alberta
October 1 – 3, 2004

Searching and Sweeping Graphs

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There are a variety of practical problems that may be modelled as a problem of looking for an intruder in a graph or directed graph. If an intruder may be located only at vertices, we call them searching problems. If an intruder may be located at vertices or along edges, we call them sweeping problems. I shall give a partial survey of what is known about searching and sweeping graphs. This is the topic of a MITACS project in which I have been involved for more than two years.

New developments in the classification of orthogonal matrices

R. Craigen
University of Manitoba

A year ago I announced my resolution of a series of conjectures that could be traced 140 years back to Sylvester about certain generalizations of Hadamard matrices. Since then, however, it has come to my attention that work on this material has been carried out independently by algebraists working on problems quite unrelated to Hadamard matrices, and apparently completely unaware of the historical connection of this material to Hadamard and Sylvester, or to classical or contemporary combinatorics. Their work has preempted some of my results by a decade.

Inverse orthogonal matrices are perhaps the most general class of objects we shall consider. An *inverse orthogonal matrix* is an $n \times n$ matrix H with nonzero complex entries such that $HH^* = nI$, where H^* is not the usual Hermitian adjoint, but a similar operation: transpose the matrix and replace every entry with its reciprocal (if all entries are complex units, this reduces to the Hermitian adjoint). Hadamard matrices are inverse orthogonal matrices whose entries are real units.

Inverse orthogonal matrices are invariant under permutation of rows and columns and multiplication of rows and columns by nonzero complex numbers. The spectrum of matrices obtained from a single matrix under these operations is its *equivalence class*. The problem is to determine the number of classes in each order and to display a complete set of class representatives, a direct generalization of the equivalence problem for Hadamard matrices.

Sylvester thought he had a full classification in 1867, but in fact the question remains open. We will discuss the history and current state of affairs, how this problem and recent work relates to questions about Hadamard matrices, generalized Hadamard matrices and finite projective planes, and some new connections of this material to quantum information theory and quantum learning.

Ramanujan Graphs: what they are, where they come from and why they are interesting

Michael Doob
University of Manitoba

Ramanujan graphs are defined in terms of the value of their second-largest eigenvalue. Despite being defined by their algebraic properties as a graph, there are many interesting relations with other areas including Number Theory, Group Theory and Computer Science. As such, these graphs, after ten years of mild interest, have become a really hot topic. In this talk we'll look at some of these relations and indicate some current directions.

On the Cayley isomorphism problem for Cayley objects of nilpotent groups of some orders

Ted Dobson
Mississippi State University

A combinatorial object X is a *Cayley object* of a group G if $\mathrm{Aut}(X)$, the automorphism group of X, contains the left-regular representation of G. That is, we view the set of vertices of X as elements of G, and X is a Cayley object of G if $G_L = \{x \to gx : g \in G\} \leq \mathrm{Aut}(X)$. In general, the *Cayley isomorphism problem* asks for necessary and sufficient conditions for two Cayley objects of a group G in some class $\mathcal{K}$ of combinatorial objects to be isomorphic. Usually, the answer will be given as an explicit list of functions such that two Cayley objects of G in $\mathcal{K}$ are isomorphic if and only if one of the functions contained in the list is an isomorphism. It is not difficult to see that for every group G and class $\mathcal{K}$ of combinatorial objects, this explicit list must contain all of the automorphisms of G. If there are no other functions on this explicit list, we say that G is a *CI-group with respect to* $\mathcal{K}$.

In 1987, Pálfy showed that a group G was a CI-group with respect to *every* class of combinatorial objects if and only if either $|G| = 4$ or G was cyclic of order n and $\gcd(n, \varphi(n)) = 1$, where φ is Euler's phi function. We remark that every group of order n is cyclic if and only if $\gcd(n, \varphi(n)) = 1$. In 1999, Muzychuk showed that if G was a cyclic group of order n that satisfies similar, but weaker arithmetical conditions, then the Cayley isomorphism problem for all such groups and every class of combinatorial objects reduces to the prime power case. That is, the explicit list of functions can be constructed if one knows the corresponding list for all cyclic groups of prime power order. In 2003, the author gave a necessary condition for the Cayley isomorphism problem for all abelian groups G whose order satisfies the same arithmetical conditions given by Muzychuk and every class of combinatorial objects to reduce to the prime power case, and gave some limited examples where the necessary condition is satisfied.

We will extend the previous results, and give a sufficient condition for the Cayley isomorphism problem for nilpotent groups whose order satisfies the same arithmetical condition to reduce to the prime power case for every class of combinatorial objects. Furthermore, we show that a much wider class of abelian groups satisfy the arithmetical conditions. In particular if G is an abelian group of order n, all Sylow subgroups of G are either elementary abelian or cyclic, and every group G of order n is nilpotent, then the Cayley isomorphism problem for every class of combinatorial objects reduces to the prime power case. We give here one of several consequences of this result: If G is an abelian group of order n such that every Sylow p-subgroup of G is elementary abelian of order at most p^4, and every group of order n is nilpotent, then G is a CI-group with respect to digraphs.

On the minimum rank of graphs

Shaun M. Fallat

University of Regina

For a given undirected graph G, the minimum rank of G is defined to be the smallest possible rank over all real symmetric matrices A whose (i, j)-th entry is nonzero whenever $i \neq j$ and $\{i, j\}$ is an edge in G. A complete combinatorial characterization for the minimum rank of a graph on n vertices seems inconceivable. However, there has been considerable and worthwhile progress on this subject over the years with many exciting advances recently. In this talk, we will survey past results, discuss current trends and new facts, and attempt to tie this concept together with other important parameters associated with certain inverse eigenvalue problems for graphs.

Notes on Hex

Ryan Hayward
University of Alberta

The game of Hex has been of particular interest to mathematicians since its invention by Piet Hein and John Nash in the 1940s. In this talk I will survey some results on Hex, ranging from a little-known combinatorial proof of the well-known property that the game cannot end in a draw to recent graph- and game-theoretic results (joint with Yngvi Bjornsson, Mike Johanson, Jack van Rijswijck, Morgan Kan, and Nathan Po) which introduce the notion of ”capturing” in Hex and allow the computer solution of arbitrary Hex game-states on small boards.

A look at graph colouring through three different lenses

Mark Kayll
University of Montana

I'll begin by introducing the chromatic and circular chromatic numbers of a digraph. After covering the basics, I'll recall a famous paradoxical theorem of Paul Erdős from 1959. This will set the stage for the presentation of a digraph analogue of Erdős' theorem. The proof uses probabilistic ideas and a (perhaps) surprising application of a fact from basic algebra. I'll conclude by introducing a related digraph homomorphism problem, currently in progress with Bojan Mohar.

Constructing Laplacian Integral Graphs

Stephen Kirkland
University of Regina

For a graph G on n vertices, its *Laplacian matrix* can be written as $L = D - A$, where A is the $(0,1)$ adjacency matrix for G, and D is the diagonal matrix of vertex degrees. It is not difficult to see that 0 must be an eigenvalue of L, and that if λ is any eigenvalue of L, then $0 \leq \lambda \leq n$. The last few years have seen a developing interest in *Laplacian integral graphs* — those graphs whose Laplacian matrices have all integer eigenvalues. In this talk we will discuss some new constructions for Laplacian integral graphs.

Group Actions on Paley Matrices

Warwick de Launey
Center for Communication Research, San Diego

Let G be a finite group; let $f : G \times G \to \{1, -1\}$ be a 2-cocycle; and let R denote the corresponding extension of G. A cocyclic weighing matrix with indexing group G, cocycle f and extension group R is a weighing matrix which is equivalent to a matrix of the form $[f(x,y)g(xy)]_{x,y \in G}$. Here g is a map from the group G to the set $\{1, -1\}$. Many of the well-known families of weighing matrices turn out to be cocylic. In particular, the Paley conference and Hadamard matrices are cocyclic.

Richard M. Stafford and I have set out the general design theory of cocyclic weighing matrices, and described an agenda for analyzing a weighing matrix which is known to be cocyclic. Put simply, one wants to completely describe all of the ways a group can act regularly on the expanded design of the weighing matrix. We have also carried out the full agenda for the Paley conference matrices and the Type I Paley Hadamard matrix, and most of the agenda for the Type II Paley Hadamard matrix.

This talk will present highlights of our work. We rely heavily on classification results from algebra, and use various combinatorial arguments to bridge the gaps.

Graph decompositions and FPGA-switch box designs

Jiping (Jim) Liu
University of Lethbridge

Switch boxes are important components in an FPGA. By modeling the routing requirements as regular hypergraphs and applying the theory of graph decompositions, we obtain a reduction design technique for switch box designs. The design method produces optimum or near optimum switch boxes.

On 2-arc-transitive graphs of dihedral groups

Dragan Marušič
Univerza v Ljubljani, Slovenija

In this lecture I will discuss some of the open problems in vertex-transitive graphs and related symmetrical combinatorial objects. In particular, I will give an account of graphs enjoying certain special types of symmetry, such as for example, the 2-arc-transitive graphs. As the main ingredient of this lecture, however, I will present a recently obtained classification of 2-arc-transitive Cayley graphs of dihedral groups and its group-theoretic consequence.

Apart from the obvious examples of even length cycles, the complete graphs of even order, the complete bipartite graphs, the complete bipartite graphs with a matching removed, the incidence/nonincidence graphs associated with the projective spaces $PG(d,q)$, $d \geq 2$, and the incidence/nonincidence graphs of the unique Hadamard design H_{11} on 11 points, the only other 2-arc-transitive Cayley graphs of dihedral groups are certain covers of complete graphs of even order.

Magic Numbers of Complete Graphs

Jenny McNulty
University of Montana

An *edge-magic total labeling* of a graph G with $v = |V(G)|$ and $e = |E(G)|$ is an injection $\lambda : V(G) \cup E(G) \to \{1,2,\ldots,v+e\}$, so that $\lambda(x)+\lambda(xy)+\lambda(y) = k$ for some constant k. A graph with an edge-magic total labeling is called *edge magic*. Kotzig and Rosa (1970) showed K_n is not edge magic for $n > 6$. It can be easily verified that the graphs K_2, K_3, K_5, K_6 are all edge magic, but K_4 is not. The *magic number* $M(n)$ of K_n is the minimum t so that $K_n + tK_1$ is edge magic. In the talk we will give new upper and lower bounds of $M(n)$, discuss the exact values of several small cases, and mention some open problems.

Colouring subsets of the Rado graph

Norbert Sauer
University of Calgary

Introduction to the Rado graph and some of its easier to develop properties. A history of colouring results of subsets of the Rado graph together with some mention of generalizations to homogeneous structures. Discussion of the most recent results, their connection to the Nesetril, Rödl structural partition theory and a very short mention of the recently discovered connection of this structural partition theory to topological dynamics.