# Ordering Techniques for ESOP-Based Toffoli Cascade Generation

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### Abstract

This paper describes three techniques for ordering ESOP cubes prior to generation of a Toffoli gate generation. Two of these techniques are from earlier work, while the third is a new approach. The new approach both reorders the cubes and manipulates them to reduce their size and complexity. Our experiments demonstrate that the new approach is much more effective than either of the two previous approaches. We apply template matching as a post-processing step, which results in even further reductions in the number of Toffoli gates.

# 1. Introduction

The motivation behind reversible logic is that the laws of physics limit the energy efficiency of traditional irreversible logic, a limit which is likely to be reached within the next 10 to 20 years. In order to advance beyond this point circuits will require an increasing amount of reversibility in their design. It was demonstrated as early as the 1960s [7] that reversible logic would be necessary for lower power dissipation in circuits. Reversible logic also has connections to quantum computing, and in fact reversible circuits can be viewed as a special case of quantum circuits [12]. One author notes that "... logic synthesis for classical reversible circuits is a first step toward synthesis of quantum circuits" [15]. Synthesis techniques for traditional irreversible logic are of little use when the goal is a reversible cascade of gates, and thus many researchers are investigating new techniques in this area.

This paper compares the results of using different cost metric functions for synthesizing Toffoli gate networks from an initial exclusive-or sum-of-products (ESOP) representation of a Boolean function as well as presenting a new method. The initial function may be reversible or irreversible; our technique ensures that the final cascade is reversible. Since the use of templates is a well-known reversible logic technique that requires the circuit to already be in reversible form, we compare results of our cost metrics both before and after applying templates.

# 2. Background

# 2.1. ESOP Synthesis

An *n*-bit generalized Toffoli gate is a reversible logic gate that has *n* inputs and *n* outputs and is described as  $(x_1, x_2, \ldots, x_n) \rightarrow (x_1, x_2, \ldots, (x_1 \cdot x_2 \cdots x_{n-1}) \oplus x_n)$ . A NOT gate is a special case of a Toffoli gate where n = 1.

An exclusive-or sum-of-products (ESOP) representation of a switching function is similar to the standard sum-of-products representation except that ORs are replaced with XORs. An ESOP can be used to represent any Boolean function [16], and a gate-level model of a reversible circuit may be generated from an ESOP by simply replacing each cube with a Toffoli gate. The straightforward method of replacing cubes with Toffoli gates would require that the circuit have 2n + m input lines, or qubits;<sup>1</sup> one for each inverted literal, one for each non-inverted literal, and one for each output. To reduce this to n + m we use a NOT gate to invert the line when needed; however with a poor ordering of the cubes this can result in an unnecessary number of gates. It is therefore important to use some technique to find a "good" ordering.

For the first two ordering methods in this work a cost metric is calculated for each input variable. The input variable with the lowest cost is chosen, and the ESOP cubes are sorted such that all cubes containing the nonnegated form of the selected variable appear before all cubes containing its negated form. This allows a single NOT gate to divide the negated and non-negated forms. Where the variable in a cube is a don't-care (-) the cube is placed in the non-negated list. The process is continued for the negated and non-negated lists until all variables have been selected.

<sup>&</sup>lt;sup>1</sup>The technically correct term is, in this area, qubits, since we assume some future quantum implementation. The reader is referred to [12] for details on quantum computing.

# 2.2. The Cost Metrics

### 2.2.1 Alpha-Beta Cost Metric

The following technique was introduced in [3].

$$\cot_{v} = \alpha \frac{1}{\sum |v_{i}|} + \beta \left| \sum v_{i} \right|$$
(1)  
where 
$$\begin{cases} v \text{ in } \operatorname{cube}_{i} \left\{ \begin{array}{l} v \text{ is positive } v_{i} = 1 \\ v_{i} = -1 \end{array} \right. \\ v_{i} = 0 \end{cases}$$

The cost metric described above is computed for each variable v in turn. The frequency sum (which is multiplied by  $\alpha$ ) determines how often the variable is used in one form or another over the entire function, while the balance sum (which is multiplied by  $\beta$ ) determines the absolute value of the difference between the number of 0s and the number of 1s assigned to that variable. Thus if a variable appears in the function relatively rarely and has close to an equal number of 0s and 1s its cost is computed to be lower. Further details of this technique are given in [3] and [13].

#### 2.2.2 AC Cost Metric

The autocorrelation (AC) transform, based on Equation 2, can be used to transform a function from the traditional Boolean domain to the spectral domain. The resulting data can be used to perform an analysis of the relative dependency of the function on its variables.

$$B^{fg}(\tau) = \sum_{v=0}^{2^n - 1} f(v) \cdot g(v \oplus \tau)$$
 (2)

The AC transform is obtained when f and g in Equation 2 are the same function. Applying the AC transform provides a value of how a function compares with itself at a "shift" of some value, given by  $\tau$ . This shift corresponds in effect to inverting the inputs corresponding to values of 1 in the binary expansion of  $\tau$ . Of the resulting AC coefficients, only first-order coefficients (where a single variable is inverted) are used in this cost metric as it allows a measurement of the function's dependency on that variable. Theoretically, a higher value for a coefficient indicates less of a dependency, thus a variable with a low coefficient should be chosen first for generating an ordering. An example is shown in Figure 1. Coefficients for multiple-output functions can be computed by applying the AC transform to each output separately and then combining the results for all outputs. Further details and examples are given in [14].

$x_3 x_2 x_1$	$\int f(X)$	au	B( au)
000	0	000	5
001	1	001	2
010	0	010	4
011	1	011	
100	0	100	4
101	1	101	
110	1	110	
111	1	111	
(a) Truth	table.	(b) Firs	t order AC coef

Figure 1: The AC coeffs for  $f(X) = x_1 \oplus x_3 x_2 \overline{x}_1$ .

# 2.3. Related Work

There are a variety of synthesis techniques for reversible logic in the literature, for instance [15, 9, 4] and [6]. We briefly mention a few of these techniques which use an ESOP or similar representation.

Gupta *et al.* [4] present a reversible logic synthesis technique based on a related representation, the positivepolarity Reed-Muller (PPRM) expansion. This work uses a tree structure to investigate all possible factors of each term, allowing the construction of a circuit that shares factors. However the PPRM representation is a special type of ESOP with a more rigorous definition, and thus will almost always have more terms than the ESOP representation used in our work.

The techniques suggested by Perkowski *et al.* in [6] and an earlier work [11] also have some relation to the general ESOP-based approach. The more recent of these works requires a factorization of each of the ESOPs representing the multiple outputs, and a new class of reversible gates is introduced. This method reported achieving good results in terms of gate numbers, but appears to require a large number of garbage outputs.

More recently Hamza and Dueck [5] have suggested a linear programming approach to ESOP cube ordering. Further comparisons to this work are given in Section 5.

#### 2.4. Template Optimization

One of our investigations involved the application of templates in order to further optimize the circuit sizes. According to [2] a template is used to replace a sequence of gates within a Toffoli network with a different sequence of gates without altering a newtork's function. In addition the replacement sequence should be smaller than the replaced sequence. Due to lack of space we direct the reader to [2] for further details.

#### **3. Proposed New Method**

The previous two ordering methods for ordering ESOP cubes in preparation for generating a Toffoli-gate cascade can be improved upon, and in this Section we propose a new method. This method has two steps: in the first step a number of rules are applied to the ESOP cube-list, and in the second step the list is then ordered to minimize the number of NOT gates. Unlike the previous two methods, in which the only improvement can be reduction of the number of NOT gates, the purpose of the following rules is to reduce the both the number of NOT gates and/or the size of the Toffoli gates.

# 3.1. Rules

For Rules 1 and 2 we consider the following cube:

$$x_1 x_2 \dots x_n \to f_1 f_2 \dots f_m \tag{3}$$

- Rule 1 If the cube contains only don't care values, *i.e.*  $x_1 = x_2 = \ldots = x_n = -$ , then for every output  $f_p = 1$ , where  $p \in \{1, 2, \ldots, m\}$ , the qubit of the corresponding output line is inverted and the cube is removed from the cube-list. Usually, output lines are initialized with a 0 qubit. The purpose of this rule is to remove the NOT gates required for this cube and complement the initial values of the corresponding output lines.
- Rule 2 If x<sub>i</sub> = 0 and x<sub>k</sub> = -, ∀k ∈ {1, 2, ..., n} {i}, then for every output f<sub>p</sub> = 1 where p ∈ {1, 2, ..., m}, the qubit of the corresponding output line is inverted and x<sub>i</sub> is set to 1. The purpose of this rule is to complement x<sub>i</sub>.



Figure 2: An example of applying Rules 1 and 2 to generate a Toffoli gate cascade from an ESOP cube-list.

An example is shown in Figure 2.

The following illustrates the labeling used for explanation of the next four rules:

Cube1 
$$x_1x_2...x_{i-1}x_ix_{i+1}...x_{j-1}x_jx_{j+1}...x_n$$
  
 $\rightarrow f_1f_2...f_{p-1}f_pf_{p+1}...f_m$   
Cube2  $y_1y_2...y_{i-1}y_iy_{i+1}...y_{j-1}y_jy_{j+1}...y_n$   
 $\rightarrow g_1g_2...g_{p-1}g_pg_{p+1}...g_m$ 

• Rule 3 If  $x_i = 0, x_j = -, y_i = -, y_j = 0, x_k = y_k$ ,  $\forall k \in \{1, 2, \dots, n\} - \{i, j\}$ , and  $f_p = g_p = 1$  for any  $p \in \{1, 2, \dots, m\}$ , then Cube1 and Cube2 can be transformed into the following four cubes.

Cube1'	$x_1 x_2 \dots x_{i-1} \ 1 \ x_{i+1} \dots x_{j-1} \ - \ x_{j+1} \dots x_n$
	$\rightarrow 00 \dots 010 \dots 0 \ (f_q = 0 \ \forall \ q \neq p)$
Cube2'	$y_1 y_2 \dots y_{i-1} - y_{i+1} \dots y_{j-1} \ 1 \ y_{j+1} \dots y_n$
	$\rightarrow 00 \dots 010 \dots 0 \ (g_q = 0 \ \forall \ q \neq p)$
Cube3	$x_1 x_2 \dots x_{i-1} \ 0 \ x_{i+1} \dots x_{j-1} \ - \ x_{j+1} \dots x_n$
	$\rightarrow f_1 f_2 \dots f_{p-1} \ 0 \ f_{p+1} \dots f_m$
Cube4	$y_1y_2\ldots y_{i-1} - y_{i+1}\ldots y_{j-1} \ 0 \ y_{j+1}\ldots y_n$
	$\rightarrow g_1 g_2 \dots g_{p-1} \ 0 \ g_{p+1} \dots g_m$

The objective of applying this rule is to complement both  $x_i$  and  $y_j$ . We note that this rule can create two more cubes, however if  $f_q = g_q = 0 \ \forall q \in \{1, 2, \dots, m\} - \{p\}$ then the last two cubes are not required since the output parts of these cubes are all zero. In order to avoid creating any new cubes unnecessarily this rule is only applied when  $f_p = g_p \ \forall p \in \{1, 2, \dots, m\}$ . An example is shown



Figure 3: Example of applying Rule 3.

in Figure 3.

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• Rule 4 [1] If  $x_i = 1$ ,  $x_j = 1$ ,  $y_i = 0$ ,  $y_j = 0$ , and  $x_k = y_k \forall k \in \{1, 2, ..., n\} - \{i, j\}$ , and  $f_p = g_p = 1$  for any  $p \in \{1, 2, ..., m\}$ , then Cube1 and Cube2 can be transformed into the following five cubes.

$x_1x_2x_{i-1} \mid x_{i+1}x_{j-1} - x_{j+1}x_n$
$\to 00\dots 010\dots 0 \ (f_q = 0 \ \forall \ q \neq p)$
$y_1 y_2 \dots y_{i-1} - y_{i+1} \dots y_{j-1} \ 1 \ y_{j+1} \dots y_n$
$\rightarrow 00\dots 010\dots 0 \ (g_q = 0 \ \forall \ q \neq p)$
$y_1y_2\ldots y_{i-1} - y_{i+1}\ldots y_{j-1} - y_{j+1}\ldots y_n$
$\rightarrow 00 \dots 010 \dots 0 \ (g_q = 0 \ \forall \ q \neq p)$
$x_1 x_2 \dots x_{i-1} \ 1 \ x_{i+1} \dots x_{j-1} \ 1 \ x_{j+1} \dots x_n$
$\to f_1 f_2 \dots f_{p-1} \ 0 \ f_{p+1} \dots f_m$
$y_1y_2\ldots y_{i-1} \ 0 \ y_{i+1}\ldots y_{j-1} \ 0 \ y_{j+1}\ldots y_n$
$\rightarrow g_1 g_2 \dots g_{p-1} \ 0 \ g_{p+1} \dots g_m$

The purpose of this rule is to reduce the size of the Toffoli gates when transforming the cubes into gates. This rule can create three extra cubes, which is not at all desirable. Like Rule 3, if  $f_q = g_q = 0 \forall q \in \{1, 2, ..., m\} - \{p\}$ , then the last two cubes are not required. For exam-

$egin{array}{c} x_1 \ 1 \ 0 \end{array}$	$x_{2}$ 1 0 (a) A	x <sub>3</sub> 1 1 n initia	$f_1$ 1 1 al cube	f <sub>2</sub> 1 1 e-list.	$\begin{array}{c} f_3 \\ 1 \\ 0 \end{array}$	x1 x2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$x_1$ - - 1	$x_2$ - 1 - 1	$egin{array}{c} x_3 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ $	$egin{array}{c} f_1 \ 1 \ 1 \ 1 \ 1 \ 0 \end{array}$	$egin{array}{c} f_2 \ 1 \ 1 \ 1 \ 1 \ 0 \end{array}$	$egin{array}{c} f_3 \ 0 \ 0 \ 0 \ 1 \end{array}$	x1 x2 x3 0 0 0 (d) Toffoli cascade for the cube-list in Fig-
(c) A (	cube-lis	st equi	valent	to Fig	ure 4a.	ure 4c.

Figure 4: Example of applying Rule 4.

ple, we can apply Rule 4 to the cubes shown in Figure 4a. Only outputs  $f_1$  and  $f_2$  contain both cubes. As the output  $f_3$  does not include the second cube, application of this rule will generate two extra cubes as shown in Figure 4c.

• Rule 5 If  $x_i = 1$  or 0,  $y_i = -$  and  $x_k = y_k$  $\forall k \in \{1, 2, ..., n\} - \{i\}$ , and  $f_p = g_p = 1$  for any  $p \in \{1, 2, ..., m\}$ , then Cube1 and Cube2 can be transformed into the following three cubes.

Cube1'	$x_1x_2\ldots x_{i-1}\overline{x}_ix_{i+1}\ldots x_{j-1}x_jx_{j+1}\ldots x_n$
	$\rightarrow 00 \dots 010 \dots 0 \ (f_q = 0 \ \forall \ q \neq p)$
Cube3	$x_1x_2\ldots x_{i-1}x_ix_{i+1}\ldots x_{j-1}x_jx_{j+1}\ldots x_n$
	$\to f_1 f_2 \dots f_{p-1} \ 0 \ f_{p+1} \dots f_m$
Cube4	$y_1y_2\ldots y_{i-1} - y_{i+1}\ldots y_{j-1}y_jy_{j+1}\ldots y_n$
	$\rightarrow g_1 g_2 \dots g_{p-1} \ 0 \ g_{p+1} \dots g_m$

This rule eliminates one cube but may create two extra cubes. If  $f_q = g_q = 0 \ \forall q \in \{1, 2, \dots, m\} - \{p\}$ , then the latter two cubes are not generated. Similar to Rule 4, this rule helps reduce the size of required Toffoli gates. For



Figure 5: Example of applying Rule 5.

example, consider the cube-list given in Figure 5a. We

apply Rule 5 to generate the modified cube-list as shown in Figure 5b, which consists of only one cube. Since two functions  $f_1$  and  $f_2$  have both cubes, one cube from the original cube-list (Figure 5a) can be removed and no more cubes are generated.

Applying the rules in different order can change the results. We have found through experimentation that applying the rules in the order 4, 5, 1, 2 and 3 produces better results for a number of benchmark circuits. Further investigation and experimentation into this is required.

### 3.2. Ordering

After applying the rules, a greedy approach is used to reorder the cubes. All the input variables initially have positive polarity. A sequence with the length of number of input variables is used to determine the current polarity of each variable. This sequence is initialized with all 1s and changed after reordering each cube (adding a NOT gate) which alters the polarity. The algorithm begins with the cube which has the least number of 0s *i.e.*, the highest number of 1s and don't care values. The sequence is then updated, and the distance between this sequence and each of the remaining cubes is calculated. The term distance refers to the Hamming Distance, or the count of the number of variables with positive polarity in one cube and negative polarity in the other. The cubes with distance zero are reordered without changing the sequence, and hence no more NOTs are needed for these cubes. Next the cube with the minimum positive distance is considered due to the requirement of the fewest possible NOT gates when this cube will be transformed into a Toffoli gate. If two or more cubes have the same distance then the cube with more don't care values is selected. The process is repeated until all cubes are processed.

# 4. Experiments

The following process was used to generate the final benchmark test results: 1. generate an ESOP representation for each benchmark using EXORCISM-4 [10]; 2. generate a Toffoli gate cascade using each of the different ordering and/or cube manipulation approaches; 3. convert the synthesized circuits into a form usable by the template optimization algorithm; and 4. perform template optimization on each of the reformatted synthesized circuits. Comparisons among the approaches were performed both before and after templates were applied. All experiments were performed on a 3.00GHz Intel(R) Pentium(R) 4 machine with 1GB RAM running CentOS release 5.3. The benchmarks used are listed in Table 2.

	avg % reduction	num. of smaller
	gate count	circuits generated
AB vs AC	18%	56 out of 77
AB+t vs AC+t	2%	47 out of 77
NM vs AB	91%	70 out of 77
NM vs AB+t	11%	61 out of 77
NM+t vs AB+t	14%	63 out of 77

Table 1: Comparisons between the various ESOP ordering and optimization methods.

# 5. Results & Discussion

The AB and AC approaches were first compared in [14], although this previous work did not report comparisons after applying template optimization. Our tests show that even with the application of templates the AB cost metric still outperforms the AC method.

The proposed new method (NM) of sorting ESOP cubes prior to generating a Toffoli gate cascade proved to be very effective. We have restricted our comparisons to the AB cost metric method, as that was the better of the prior two methods tested. For these comparisons we use the best result generated by all values of alpha for each of the 77 benchmarks. Without applying template optimization, the NM resulted in a 90% average reduction in gates, and generated a smaller circuit on 70 out of 77 of the benchmarks tested. After applying template optimization to both methods we still obtained improvements of an average 81% reduction in gates and a smaller circuit on 63 out of 77 benchmarks. Table 1 summarizes the results. The notation "+t" indicates that template optimization was used as a post-processing step. It is interesting to note that template optimization is able to reduce 67 of the benchmarks by an average of 19 gates when applied after the AB method, while for the new method, template optimization only improves the resulting circuit for 40 benchmarks, and for those 40 there is an average reduction of 5 gates.

We briefly discuss our work in comparison to the work by Dueck and Hamza [5] which also looks at a more effective ordering approach combined with attempting to reduce cube sizes. The most significant difference lies in our pre-application of rules to minimize the number and sizes of the Toffoli gates, which does not take place in the Dueck and Hamza approach. It is difficult to make accurate comparions between their work and ours as there is a relatively small overlap of benchmarks, and we also suspect that benchmarks that appear to be the same are not necessarily so. Future work will involve generating a common list of benchmarks, likely from the Revlib library [17], in order to carry out a thorough comparison between our two efforts. In order for the readers to examine our results for themselves we have included in Table 2 the list of benchmarks used in this work, as well as both gate count and quantum cost results. The quantum costs are based on values for Toffoli gates as given in [8];

# 6. Conclusions and Future Work

This paper introduces a new technique for manipulating and ordering ESOP cubes prior to generating a cascade of Toffoli gates from the ESOP list. In addition we apply template optimization to results from our new technique as well as two previously published techniques, in order to determine whether any technique might be better or worse suited to post-application of template optimization. We found that the AC cost metric is generally the least effective, while the AB cost metric with  $\alpha = 0$  was more effective. Application of templates improved the results from both of these approaches, but we still found the AB results to be better than the AC results. However, the proposed new rule-based method achieved better results than even of the two previously introduced methods, before and after templates were applied. Work on the new proposed method is still in its preliminary phases, and can benefit from further experimentation and in particular comparisons to results such as those reported by [4, 6] and in particular [5].

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<b>D</b> escribed	1	AC	AB	]	NM
Function	inputs	GC	GC	GC	OC
av2	6	12	13	11	160
ex2	0	12	15	11	100
ex3	0	/			97
majority	6	8	8	8	125
xor5	6	7	7	5	5
C17	7	9	9	9	105
0000	6	17	22	10	142
cillo2a	0	17	10	19	145
12	8	22	19	20	246
rd53	8	24	27	27	269
con1	9	19	21	23	206
Qeum	10	122	120	111	6501
9 sym	10	132	127	111	0501
9symm1	10	132	129	111	6501
life	10	114	107	92	3711
life min	10	114	107	92	3711
max/6	10	115	107	80	4068
1122	10	115	107	70	1150
ra/3	10	85	80	13	1150
sqn	10	91	76	63	1675
dc1	11	36	39	37	454
sym10	11	197	193	162	10358
symmetry	11	20	25	20	219
wiili	111	50	25	20	210
z4	11	50	48	48	642
z4ml	11	50	48	48	642
cm152a	12	15	16	15	215
rd84	12	134	111	00	2558
1004	12	1.54	111	20	616
sqrt8	12	4/	40	38	010
adr4	13	46	55	34	630
dist	13	237	185	170	7210
radd	13	36	48	51	669
raat	12	142	00	00	2202
1001	15	142	99	00	3393
squar5	13	30	43	41	465
clip	14	195	174	148	6570
cm42a	14	35	35	20	270
000850	14	75	60	64	2222
CIII6Ja	14	15	25	04	2222
pm1	14	35	35	20	270
sao2	14	129	88	77	7495
co14	15	30	30	30	3488
dc2	15	103	75	65	1778
micor 1	15	62	55	55	1012
misexi	15	05	55	55	1012
alu2	16	184	157	136	5118
alu2	15	184 AC	AB	136	5118
alu2 Function	15 16 inputs	05 184 AC GC	AB GC	136 136	5118 NM OC
Function	16 inputs	AC GC 184	AB GC 157	136 136	5118 NM QC 5118
Function example2	15 16 16 16	AC GC 184 184 118	AB GC 157 93	136 136 GC 136 84	NM QC 5118 1969
Function example2 inc	15 16 16 16	AC GC 184 118 118	AB GC 157 93	136 136 GC 136 84 122	1012 5118 <u>VM</u> <u>QC</u> 5118 1969 2500
Function example2 inc mlp4	15 16 16 16 16 16	AC GC 184 118 118 151	AB GC 157 93 131	136 136 136 84 122	NM QC 5118 1969 3500
Function example2 inc mlp4 5xp1	15 16 16 16 16 16 17	AC GC 184 184 118 151 102	AB GC 157 93 131 85	136 136 136 84 122 75	IO12           5118           NM           QC           5118           1969           3500           1229
Function example2 inc mlp4 5xp1 parity	15 16 16 16 16 16 17 17	AC GC 184 184 118 151 102 32	AB GC 157 93 131 85 32	136 136 GC 136 84 122 75 16	IO12           5118           VM           QC           5118           1969           3500           1229           16
Function example2 inc mlp4 5xp1 parity ryy6	13 16 inputs 16 16 16 17 17 17 17	05           184           AC           GC           184           151           102           32           44	AB GC 157 93 131 85 32 44	136 136 GC 136 84 122 75 16 44	IO12           5118           VM           QC           5118           1969           3500           1229           16           4298
Function example2 inc mlp4 5xp1 parity ryy6 t481	13 16 inputs 16 16 16 16 16 17 17 17 17 17	05           184           AC           GC           184           151           102           32           44           21	33           157           AB           GC           157           93           131           85           32           44           21	136 136 GC 136 84 122 75 16 44 20	IO12           5118           NM           QC           5118           1969           3500           1229           16           4298           236
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2	13           16           inputs           16           16           16           17           17           17           17           17           17           17           17	AC GC 184 118 151 102 32 44 21 49	AB GC 157 93 131 85 32 44 21 38	136 136 136 84 122 75 16 44 20 30	1012           5118           NM           QC           5118           1969           3500           1229           16           4298           236           576
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3	13           16           inputs           16           16           17           17           17           17           17           17           17           17           17           17	03           184           AC           GC           184           118           151           102           32           44           21           49           96	33 157 AB GC 157 93 131 85 32 44 21 38 94	136 136 136 84 122 75 16 44 20 30 81	1012           5118           QC           5118           1969           3500           1229           16           4298           236           5776           2346
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27	15           16           inputs           16           16           17           17           17           17           18	AC         GC           184         118           151         102           32         44           21         49           96         31	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24	136 136 136 84 122 75 16 44 20 30 81 24	5118 <u>QC</u> 5118 1969 3500 1229 16 4298 236 576 2346 252
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqrf	13           16           inputs           16           16           17           17           17           17           17           17           17           17           17           18           18           17	AC         GC           184         184           118         151           102         32           44         21           49         96           31         66	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81	136 136 136 84 122 75 16 44 20 30 81 24 69	5118 5118 <u>QC</u> 5118 1969 3500 1229 16 4298 236 576 2346 252 1030
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6	13           16           inputs           16           16           16           17           17           17           17           17           17           17           18           18           17           19	03           184           AC           GC           184           118           151           102           32           44           21           49           96           31           66           146	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81	136 136 136 84 122 75 16 44 20 30 81 24 69 9197	5118 5118 <u>VM</u> 2C 5118 1969 3500 1229 16 4298 236 2346 252 1030 576 2346
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 che1	15           inputs           16           16           17           17           17           17           17           17           17           17           17           17           17           17           17           17           17           17           17           17           20	03           184           AC           GC           184           118           151           102           32           44           21           49           96           31           66           146	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           229	136 136 136 84 122 75 16 44 20 30 81 24 69 197 20	Still           5118           NM           QC           5118           1969           3500           1229           16           4298           236           5776           2346           252           1030           5757           230
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1	13           16           inputs           16           16           17           17           17           17           17           17           19           20	AC         GC           184         AC         GC           184         18         151           102         32         44           21         49         96           31         66         146           37         40	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           229           32	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29	NM         QC           5118         1969           3500         1229           16         4298           236         576           2346         252           1030         5757           239         239
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 adu1 cmb	15           inputs           16           16           16           17           17           17           17           17           17           17           17           17           17           17           18           18           19           20           20           20           20           20	03           184           AC           GC           184           151           102           32           44           21           49           96           31           66           146           37           400	35           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           229           32           18	136 136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16	NM           QC           5118           1969           3500           1229           16           4298           236           5776           2346           576           2352           1030           5757           239           908           4208
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010	15           inputs           16           16           16           17           17           17           17           17           20           20	03           184           118           151           102           32           44           21           49           96           31           66           146           37           40           2286	33           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           229           18           2611	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           252           1030           5757           239           908           126108
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 alu3 dk27 sqr6 alu4 cmb ex1010 C7552	15           inputs           16           16           16           17           17           17           17           17           19           20           21	AC           GC           184           118           151           102           32           44           21           49           96           146           37           40           2286           47	33 157 AB GC 157 93 131 85 32 44 21 38 94 24 81 229 32 32 38 94 81 229 32 38 94 81 81 83 85 85 85 85 85 85 85 85 85 85	136 136 GC 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31	NM           QC           5118           1969           3500           1229           6           4298           236           5776           2346           252           1030           5757           239           908           126108           673
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 adu1 cmb ex1010 C7552 decod	15           16           inputs           16           16           16           17           17           17           17           17           18           18           17           19           20           20           20           20           21	03           184           184           118           151           102           32           44           21           49           96           31           66           146           37           40           2286           47           47	35           157           93           131           85           32           44           21           38           94           24           81           229           32           18           2611           80	136 GC 136 84 122 75 16 44 20 30 81 24 69 197 29 165 31 31	NM           QC           5118           1969           3500           1229           16           4298           236           5776           236           576           236           576           239           908           126108           673
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17	15           inputs           16           16           16           17           17           17           17           18           18           17           20           20           20           21           21	AC           GC           184           118           151           102           32           44           21           49           96           146           37           40           2286           47           57	33 157 AB GC 157 93 131 85 32 44 21 38 94 24 81 229 32 18 2611 80 80 49	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 45	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           252           1030           5757           239           908           126108           673           673           1465
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 alu3 dk27 sqr6 alu4 cmb ex1010 C7552 decod dk17 pcler8	15           16           inputs           16           16           17           17           17           17           18           18           19           20           20           21           21           21	03           184           AC           GC           184           118           151           102           32           44           21           49           96           31           66           146           37           40           2286           47           57           31	AB GC 157 93 131 85 32 44 21 38 94 24 81 229 32 18 80 80 49 22	136 136 84 122 75 16 44 20 30 81 24 69 197 29 197 29 1965 31 31 31 45	NM           QC           5118           1969           3500           1229           16           4298           236           5776           2346           522           1030           5776           239           908           126108           673           673           1465           340
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4	15           16           inputs           16           16           16           17           17           17           18           17           19           20           20           21           21           21           21           21           22	03           184           184           118           102           32           44           21           49           96           146           140           2286           47           47           57           31           1414	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           92           1063	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 45 31 45 19 842	NM           QC           5118           1969           3500           1229           16           4298           236           5776           236           5776           236           5757           239           908           126108           673           1465           3400           47844
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 anla	15           16           inputs           16           16           16           17           17           17           18           18           19           20           21           21           21           21           21           22           22	AC           GC           184           118           151           102           32           44           21           49           96           31           66           146           37           40           2286           47           57           31           1414	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           22           18           2611           80           49           22           1063	136 136 136 84 122 75 16 44 20 30 81 20 30 81 24 69 197 29 16 1965 31 31 31 31 9 842 270	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           5757           239           908           126108           673           673           340           478844           2884
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla err150e	15           16           inputs           16           16           17           17           17           17           17           17           17           18           18           17           19           20           20           21           21           21           21           21           21           22           22           22           22	03           184           184           1181           102           32           44           21           49           96           166           137           40           2286           47           47           57           11414           104	157           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           224           18           2611           80           49           22           1063           80           82	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 45 31 31 45 9 842 70	NM           QC           5118           1969           3500           1229           16           4298           236           5716           2346           572           1030           5757           239           908           126108           673           47844           2884           282
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a e51 ro	15           inputs           16           16           16           17           17           17           17           17           18           17           19           20           20           20           21           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           23	03           184           184           1102           32           44           21           49           96           166           146           17           31           66           146           47           57           31           141           104           53           877	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           21           38           94           22           18           2611           80           49           22           1063           80           53           62	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 45 19 842 70 49	NM           QC           5118           1969           3500           1229           16           4298           236           5776           236           576           236           5757           239           908           126108           673           1465           3400           47844           2884           833           2404
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 alu3 dk27 sqr6 alu4 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m	15           16           inputs           16           16           17           17           17           17           17           18           18           19           20           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22	03           184           184           1181           102           32           44           21           49           66           146           37           400           2286           47           47           57           31           1414           104           53           877	357           157           157           93           131           85           32           44           21           38           94           24           81           2611           80           80           92           1063           80           53           663	136 136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 165 31 31 31 45 555 55 55	NM           QC           5118           1969           3500           1229           6           4298           236           576           2346           2346           2346           2346           2346           2346           5757           239           908           126108           673           440           47844           2833           34084
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux mux	15           16           inputs           16           16           16           17           17           17           17           17           17           17           17           17           17           12           20           20           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           23	03           184           184           118           102           32           44           21           49           96           166           144           21           49           96           166           140           2286           47           47           47           47           47           47           47           47           57           31           1414           104           53           877           35	357           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           224           18           2611           80           49           22           1063           80           53           663           35	136 136 136 84 122 75 16 44 20 30 81 24 69 1965 31 45 19 19 65 31 45 19 842 70 45 31 45 31 45 31 45 31 45 31 31 45 31 45 31 45 31 31 45 31 31 31 31 31 31 31 31 31 31	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           576           2346           575           239           908           126108           673           1465           3404           47844           833           34084           815
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial	15           inputs           16           16           16           17           17           17           17           18           17           20           20           20           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22	03           184           184           118           151           102           32           44           21           49           96           166           146           37           40           2286           47           57           311           104           53           877           35           1426	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           22           18           2611           80           92           1063           80           53           663           35           1041	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 31 45 19 842 70 49 842 70 49 555 31 833	NM           QC           5118           1969           3500           1229           16           4298           236           5776           2346           576           2346           5757           239           126108           673           1465           3400           47844           833           34084           815           49189
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 alu3 dk27 sqr6 alu4 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12	15           inputs           16           16           16           17           17           17           17           18           18           17           19           20           21           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           24	03           184           184           1181           102           32           44           21           49           66           146           37           40           2286           47           47           57           31           1414           104           53           1426           53           1414           104           53           1426           53           1426           53           1426           53           1426           53           1426           53           1426           53           1426           53           1426           54           51           51           51           51           51           51           51           51           51      51 <t< td=""><td>357           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           2611           80           40           22           1063           80           53           663           35           1043           62</td><td>136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 31 45 55 31 842 70 49 9 555 31 842 70</td><td>NM           QC           5118           1969           3500           1229           16           4298           236           5716           2346           2352           10307           239           908           126108           673           673           3400           47844           2884           815           91259</td></t<>	357           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           2611           80           40           22           1063           80           53           663           35           1043           62	136 136 136 84 122 75 16 44 20 30 81 24 69 197 29 16 1965 31 31 31 45 55 31 842 70 49 9 555 31 842 70	NM           QC           5118           1969           3500           1229           16           4298           236           5716           2346           2352           10307           239           908           126108           673           673           3400           47844           2884           815           91259
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 adu6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12 cordic	15           16           inputs           16           16           17           17           17           17           17           18           18           17           20           20           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           23	03           184           184           118           102           32           44           21           49           96           146           31           66           146           37           40           2286           47           47           47           47           57           11414           104           53           1426           74           35           1426           74           3045	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           24           81           224           80           49           32           18           2611           80           49           22           1063           80           53           663           35           1041           22           2533	136 136 136 84 122 75 16 44 20 30 81 24 69 1965 31 45 19 842 70 45 31 842 70 45 55 31 835 51790	NM           QC           5118           1969           3500           1229           16           4298           236           576           234           235           1030           5757           239           908           126103           673           1465           34084           815           49189           1259           348779
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12 cordic cu	15           16           inputs           16           16           16           17           17           17           18           18           19           20           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           23	03           184           184           118           151           102           32           44           21           49           96           1166           146           37           40           2286           47           57           311           1414           53           877           1426           74           3045           49	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           24           81           2292           18           2611           80           53           663           35           1041           62           2533           40	136 136 136 136 136 14 122 75 16 44 20 30 81 24 69 197 29 16 6 1965 31 31 31 31 31 31 31 555 31 33 55 1790 39	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           5757           239           9008           126108           673           1465           340           47844           2833           34084           815           91299           348799           348799           348799           348799           348799           348799           348799           348799           348799           348794
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12 cordic cu gary	15           inputs           16           16           16           17           17           17           17           17           17           17           17           17           12           20           20           21           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           23           24           25           26	03           184           184           118           151           102           32           44           21           49           96           166           17           40           2286           47           47           57           11414           104           53           1426           335           1426           573           1424           503	357           157           AB           GC           157           93           131           85           32           44           21           38           94           24           81           224           18           2611           80           492           1063           80           53           663           35           1041           62           2533           40	136 136 136 84 122 75 16 44 20 30 81 24 69 196 1965 31 31 45 19 842 70 49 197 19 842 70 31 842 70 35 1790 3285	NM           QC           5118           1969           3500           1229           16           4298           236           576           2346           576           239           908           126108           673           673           47844           2884           813           34084           815           9189           1252           348779           964           17407
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 add6 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12 cordic cu gary in0	15           inputs           16           16           16           17           17           17           17           18           17           19           20           20           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           24           25           26           26           26	03           184           184           118           102           32           44           21           49           96           1146           118           102           32           44           21           49           96           1146           37           40           2286           47           47           57           31           1414           104           53           877           31           1414           104           53           877           31           40           2286           47           47           53           877           31           1414           104           53           877           3045           49           503	357           157           AB           GC           157           93           131           85           32           44           21           38           94           21           38           94           21           38           94           21           38           94           21           32           18           2611           80           49           21           1063           80           53           1041           623           40           338           328	136 136 136 84 122 75 16 44 20 30 81 24 69 1965 31 45 1965 31 45 1965 31 45 1982 70 49 842 70 31 833 55 1790 39 285	NM           QC           5118           1969           3500           1229           16           4298           236           5776           236           5776           237           1030           5757           239           908           126108           673           1465           3400           47844           813           34084           815           49189           12579           964           17407           72497
Function example2 inc mlp4 5xp1 parity ryy6 t481 x2 alu3 dk27 sqr6 alu3 dk27 sqr6 alu3 dk27 sqr6 alu4 alu1 cmb ex1010 C7552 decod dk17 pcler8 alu4 apla cm150a f51m mux tial b12 cordic cu gary in0 role	15           16           inputs           16           16           17           17           17           17           17           18           18           19           20           21           21           21           21           21           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           22           24           25           26           26           26           26           26           26           26           26           26	03           184           184           1181           102           32           44           21           49           66           31           66           146           37           40           2286           47           47           53           1414           104           53           877           35           1426           53           674           3045           49           503           503           503	357           157           157           93           131           85           32           44           21           38           32           18           2611           80           80           92           1063           80           53           663           35           1041           62           2533           40           338           338           338           338	136 136 136 136 14 122 75 16 44 20 30 81 24 69 197 29 16 5 31 31 31 31 45 55 31 31 842 70 49 555 31 31 842 70 9 842 70 9 195 31 31 84 84 84 84 84 84 84 84 84 84 84 84 84	NM           QC           5118           1969           3500           1229           6           4298           2366           5766           2346           2350           5757           2399           908           126108           673           673           440           813           34084           815           49189           1259           348779           348779           7407           17407
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Table 2: Benchmarks used, along with results from each of the AC, AB, and NM approaches. All results are after template matching.

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