Basic Rules of Algebra for real numbers

Assume a, b, c, d are real numbers and that m, n are positive integers.

Commutativity	$a + b = b + a$	$a \cdot b = b \cdot a$
Note:	$a + a = 2a$	$a \cdot a = a^2$
Associativity	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive law	$a(b + c) = ab + ac$	
Note:	$(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd$	

Factoring Special Polynomials:

Perfect Squares	$a^2 - 2ab + b^2 = (a - b)^2$	$a^2 + 2ab + b^2 = (a + b)^2$
Difference of Squares	$a^2 - b^2 = (a - b)(a + b)$	
Difference/Sum of Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Binomial Equation: $(a + b)^n = \Sigma_{r=0}^{n} \binom{n}{r} a^{n-r} b^n$ where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ and $m! = m(m-1)\ldots 1$ also $0! = 1$

Identities and Inverses:

$a + 0 = a = 0 + a$ $\qquad$ $a \cdot 1 = a = 1 \cdot a$

$a + (-a) = 0 = (-a) + a$ $\qquad$ $a \cdot \left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right) \cdot a, a \neq 0$

Fractions:

$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$ $\qquad$ $\frac{a}{b} \cdot \left(\frac{c}{d}\right) = \frac{ac}{bd}$

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ $\qquad$ $\frac{a}{b} \cdot \left(\frac{c}{b}\right) = \frac{ac}{b^2}$

$a + \frac{b}{c} = \frac{ac}{c} + \frac{b}{c} = \frac{ac+b}{c}$ $\qquad$ $a \cdot \left(\frac{b}{c}\right) = \frac{a}{1} \cdot \left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d} = \frac{ad-bc}{bd}$ $\qquad$ $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Exponents:

$a^0 = 1, a \neq 0$ $\qquad$ $0^a = 0, a \neq 0$

$\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} = a^m$ $\qquad$ $a^{\frac{1}{m}} = \sqrt[m]{a}$

a^b is well defined when a and b are not both 0
(in other words 0^0 is not well defined for the purposes of 1st and 2nd year math courses)

a^b is a complex number if $a < 0$ and $b = m\left(\frac{1}{2^n}\right)$
(in other words the even root of a negative number is not a real number)

$a^b \cdot a^c = a^{b+c}$ $\qquad$ $\frac{a^b}{a^c} = a^{b-c}$ $\qquad$ $\left(a^b\right)^c = a^{bc}$

$(ab)^c = a^c \cdot b^c$ $\qquad$ $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$ $\qquad$ $\left(\frac{a}{b}\right)^{-c} = \frac{a^{-c}}{b^{-c}} = \frac{b^c}{a^c} = \left(\frac{b}{a}\right)^c$

$a^b + a^b = 2a^b$ $\qquad$ but $a^b + a^c$ and $a^b + c^b$ cannot be simplified

Logarithms:

If $a = b^c$ then $c = \log_b a = \frac{\ln a}{\ln b}$, $b > 0$ and $b \neq 1$.

$\log_b b^c = c$ $\qquad$ $\log_b(a^c) = c \log_b a$

$\log_b(ac) = \log_b a + \log_b c$ $\qquad$ $\log_b\left(\frac{a}{c}\right) = \log_b a - \log_b c$

$\log_b(a + a) = \log_b(2a)$ $\qquad$ but $\log_b(a + c)$ cannot be simplified