Open Problems from CCCG 2001

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The following is a list of the problems presented on August 13, 2001 at the open-problem session of the 13th Canadian Conference on Computational Geometry held in Waterloo, Canada. One problem posed at the session was later withdrawn when it became clear subsequently that it was already solved in the literature.

Matching Problem on Pseudoline Segments
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Suppose we are given

1. an arrangement of n pseudolines;
2. $n+1$ vertical lines defining n vertical slabs (excluding the extreme infinite slabs); and
3. two points, one on the leftmost vertical line, and one on the rightmost vertical line.

Refer to Figure 1. A *monotone matching* is a set of n segments, each the portion of a unique pseudoline, and each spanning a unique slab, such that the left endpoint of each segment is above the right endpoint of the segment in the previous slab. In addition, the point on the first vertical line is below the left endpoint of the first segment, and the point on the last vertical line is above the right endpoint of the last segment.

What is the complexity of finding a monotone matching, or reporting that one does not exist? Under what conditions do they exist?

Examples in which monotone matchings do and do not exist are available on the web [Str02]. This problem is in fact motivated by a question on floodlight illumination, and is stated in [Str94].

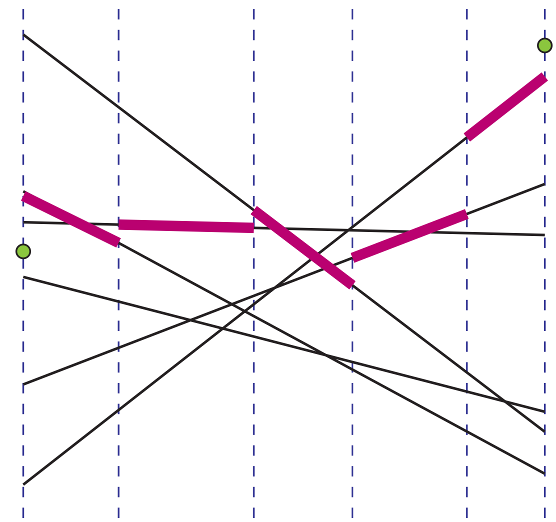

Figure 1: An example of a monotone matching.

References

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[Str02] Ileana Streinu. A matching problem for pseudoline segments. `http://cs.smith.edu/~streinu/Research/Problems/ArrangMatch/problem.html`

Proximate Point Location
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Under what conditions can a planar decomposition be preprocessed in order to support a sequence of point location queries such that a query is faster if it is "near" the previous query? (We might say that such a data structure enjoys the "dynamic finger property.") More precisely, if the sequence of queries is $q_1, \ldots, q_m$, then the time to perform query q_i should be proportional to the logarithm of their "distance," denoted $d(q_i, q_{i-1})$. Two issues remain to be specified precisely: what type of planar decomposition is supported, and what function d measures the distance between two queries. The distance function might involve both geometry (e.g., how many features are contained in an

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ellipse whose foci are the two points) as well as combinatorics (e.g., the graph distance in the dual of the planar decomposition).

Splay trees [ST85] or level-linked trees [BT80] achieve analogous results for the much simpler one-dimensional setting. Motivated by this two-dimensional problem, related results appear in CCCG 2002 [DIL02] for the variation in which the goal is to preprocess a set of points to support exact point searches in the time specified above. In this context, only a certain class of distance functions are feasible; an example is the ellipse-based distance mentioned above.

References

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Bar-Magnet Polyhedra

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[This problem appears as Problem 32 in The Open Problem Project, `http://www.cs.smith.edu/~orourke/TOPP/P32.html`.]

Which polyhedra are bar-magnet polyhedra? For reasons detailed below, the problem can be phrased as asking which 3-connected planar graphs may have their edges directed so that the directions "alternate" around each vertex.

Let P be a polyhedron with a set of edges E. For an edge $e \in E$, define a *bar magnet* as a mapping of e to either (N, S) or (S, N), which assigns the endpoints of e opposite poles of a magnet (and corresponds to directing the edge). Call a vertex v of P to be *alternating* under mappings of its edges to bar magnets if the incident edges assigns alternating magnetic poles to v in the cyclic order of those edges on the surface around v: $(N, S, N, S, ...)$. Thus if $\deg(v)$ is even, the poles alternate, and if $\deg(v)$ is odd, at most two like poles are adjacent in the circular sequence. Finally, call a polyhedron a *bar-magnet polyhedron* if there is a bar-magnet assignment of each of its edges so that each of its vertices is alternating.

Updates. At the presentation of the problem, Therese Biedl proved that the polyhedron formed by gluing together two tetrahedra with congruent bases is not a bar-magnet polyhedron: alternation at the three degree-4 vertices of the common base forces some other edge to be directed both ways. Thus not all polyhedra are bar-magnet polyhedra. Erik Demaine proved that a polyhedron all of whose vertices have even degree is a bar-magnet polyhedron: the graph has a face 2-coloring, and the edges of the faces of color 1 can oriented counterclockwise, which then orients each face of color 2 clockwise. He also observed that if every vertex is of degree 3, Petersen's theorem yields a perfect matching that establishes such "simple" polyhedra are bar-magnet polyhedra. It remains open to characterize those polyhedral graphs (or more generally, planar graphs) that may be directed to satisfy alternation.

If one (considerably) loosens the criterion to only demand "balance" at each vertex (rather than alternation), then every graph may be balanced. Define a node of a graph to be *balanced* if the number of N- and S-poles differ by at most one there; equivalently, if the number of in- and out-edges incident to the node differ by at most one. Then any graph G may be directed so that it is balanced. One can prove this by repeatedly directing and then deleting cycles, and finally balancing the remaining trees. (Proof by the Smith Problem Solving Group, Oct. 2001.)

More Pseudotriangulations than Triangulations?

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[This problem appears as Problem 40 in The Open Problem Project, `http://www.cs.smith.edu/~orourke/TOPP/P40.html`.]

For a planar point set S, is the number of pseudotriangulations always at least the number of triangulations, with equality only when S is in convex position?

A *pseudotriangle* is a planar polygon with exactly three convex vertices. Each pair of convex vertices is connected by a reflex chain, which may be just one segment. (Thus, a triangle is a pseudotriangle.) A *pseudotriangulation* of a set S of n points in the plane is a partition of the convex hull of S into pseudotriangles using S as a vertex set. A minimum pseudotriangulation, or *pointed pseudotriangulation*, has the fewest possible number of edges for a given set S of points.

The conjecture ("yes") has been established for all sets of at most 10 points: ≤ 9 by [BKPS01], and 10 by Oswin Aichholzer [personal communication, 28 Mar. 2002]. See [Str00, KKM$^+$01, O'R02] for examples, explanation of the term "pointed," and further details. In CCCG 2002, Aichholzer et al. [AAKS02] establish that the number of pointed pseudotriangulations on n points is minimized when the points are in convex position.

References

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Morse theory with two parameters?

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In topology, Morse theory studies the behavior of a function $f : X \rightarrow \mathbb{R}$ by considering level sets $L(d) = \{x \in X | f(x) = d\}$. One example application is the *contour tree*, which can be used to draw level sets (also known as *isosurfaces*). As Pascucci describes in [Pas01], the contour tree is obtained by contracting each connected component of an isosurface to a point. We can use the contour tree, therefore, to determine a "seed point" on each connected component from which that component can be traced out. The size of this tree is the number of topological changes in isosurfaces.

Suppose that we have the output of a simulation as a time-varying scalar field $f(x, y, z, t)$, and we would like to view it by considering isosurfaces $I(t, d) = \{(x, y, z) \mid d = f(x, y, z, t)\}$. Can we create a two-parameter version of the contour tree that, given t and d, will determine "seed points" for isosurfaces $I(t, d)$? In general, what can we say about Morse theory for two parameters?

References

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Smallest Sphere Intersecting Lines

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Given a set of n lines in 3-space, how quickly can we find the smallest sphere that intersects all of the lines? The same question may be asked for rays. Four lines define a sphere, so the sphere can be found in $O(n^5)$ time. Is there a better way?

In two dimensions, and indeed in higher dimensions for the analogous problem of finding the smallest ball intersecting hyperplanes, the problem can be solved in linear time [BJMR91] by linear programming in constant dimension.

References

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Sigmoid Curve Fitting Problem
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The motivation for this problem comes from our work on edge detection. We want to detect edges with "ramp profiles." The typical cross-section of an image around a ramp edge is a signal that has a "sigmoid shape," which we define as any curve that is monotonic non-decreasing, starting and ending with slope 0, with unimodal derivative.

The problem is: Given a finite signal $f : [-w, +w] \to \mathbb{R}$, fit the best sigmoid curve $s : [-w, +w] \to \mathbb{R}$ to this signal. As a start, "best" can be defined by any reasonable measure (e.g., the integral of the squared difference).

Often curve fitting problems ask for the best fit selected from a parametric family. But we do not assume any finitely parameterizable family of sigmoid curves. Another alternative is to propose a reasonable parametric family of sigmoid functions.

The non-parametric version seems more difficult. To make it tractable, let us assume the discrete version. So f is defined only on the integer values from $-w$ to $+w$. Now, a discrete sigmoid function s has the property that it is monotonic and its second-order difference is unimodal. This supports a search for all such functions using dynamic programming, but this is at best $\Theta(n^4)$, and perhaps as slow as $\Theta(n^5)$ time.

Vertex-Unfolding of Nonsimplicial Polyhedra
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[This problem appears as Problem 42 in The Open Problem Project, `http://www.cs.smith.edu/~orourke/TOPP/P42.html`.]

Consider a polyhedron with simply connected facets (no holes on a facet) and without boundary (every edge is incident to exactly two facets). Can the polyhedron be cut along potentially all of its edges, but leaving certain faces connected at vertices, and unfolded into one piece in the plane without overlap? Such an unfolding is called a *vertex-unfolding*, to distinguish from widely studied *edge-unfoldings* and *general unfoldings*. An important subproblem here is whether all convex polyhedra have vertex-unfoldings; a negative answer would also resolve whether all convex polyhedra have edge-unfoldings.

It is known that all simplicial polyhedra have vertex-unfoldings [DEE+02]. These vertex-unfoldings have a special structure called a "facet path" which does not exist in general, even for convex polyhedra [DEE+02].

References

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Embedding Triangles in 4-space
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How many triangles can be embedded with n points in 4-space? The triangle vertices must be selected among the n points, and the triangles must have pairwise disjoint interiors. It is known that the bound is $\Theta(n^2)$ in 3-space, and $\Theta(n^3)$ in $\mathbb{R}^d$ for $d \geq 5$. I conjecture that the right bound is $\Theta(n^2)$ in 4-space.