Homework 10: Due April 12th (Friday)

(1) Let
$$A = \begin{bmatrix} 1 & 4 \\ 0.5 & 0 \end{bmatrix}, \vec{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$
(a) Evaluate $A\vec{v}_1$ and $A\vec{v}_2$.
(b) Evaluate $A^{10}\vec{v}_1$ and $A^{10}\vec{v}_2$.
(c) Find $c_1$ and $c_2$ so that
$$c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$
(d) Evaluate $A^{10}\vec{w}$.
(e) Evaluate $A^{10}\vec{e}_1$ and $A^{10}\vec{e}_2$. (As usual, $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.)
(f) Evaluate $A^{10}$.

(2) Let
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$
(a) Calculate $A\vec{v}_1$, $A\vec{v}_2$, and $A\vec{v}_3$.
(b) Calculate $A^k\vec{v}_1$, $A^k\vec{v}_2$, and $A^k\vec{v}_3$.
(c) Find $c_1$, $c_2$, and $c_3$ such that
$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
(d) Calculate $A^k \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(3) Let $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$.
(a) Find nonzero vectors $\vec{v}$ and $\vec{w}$ so that $(A-3I_2)\vec{v} = 0$ and $(A+2I_2)\vec{w} = 0$.
(b) Show that $A\vec{v} = 3\vec{v}$ and $A\vec{w} = -2\vec{w}$.

(4) Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.
(a) Find eigenvalues of $A$, i.e. find real numbers $\lambda_1$ and $\lambda_2$ such that $(A - \lambda_i I_2)$ is not invertible.
(b) Find eigenvectors of $A$, i.e. find nonzero vectors $\vec{v}_1$ and $\vec{v}_2$ such that $A\vec{v}_i = \lambda_i\vec{v}_i$.
(c) Let $S = [\vec{v}_1\ \vec{v}_2]$. Calculate $S^{-1}$.
(d) Calculate $M = S^{-1}AS$.

(5) Calculate $\det(A)$ where
$$A = \begin{bmatrix} 2 & -4 & 5 \\ 0 & -3 & 6 \\ 4 & 5 & 7 \end{bmatrix}.$$