Homework 2: Due Feb 1st (Friday)

(1) Draw the following vectors in standard position (i.e. starting at the origin)

(a) $\vec{\mathbf{a}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) $\vec{\mathbf{b}} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(c) $2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}$

(2) Let $\vec{\mathbf{u}} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and let $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

(a) Solve for $\vec{\mathbf{x}}$ where

$$2\vec{\mathbf{x}} + \vec{\mathbf{u}} = 3\vec{\mathbf{v}}.$$

(b) Decide if $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a linear combination of $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ or not.

(c) Decide if $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ is a linear combination of $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ or not.

(d) Decide if $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a linear combination of $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ or not.

(3) Determine $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$ and projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$, where

$$\vec{\mathbf{u}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{\mathbf{v}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

(4) Determine $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$ and the projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$, where

$$\vec{\mathbf{u}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{\mathbf{v}} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$$

(5) Show that for any pair of vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ in $\mathbb{R}^n$ we have

$$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}}) = \|\vec{\mathbf{u}}\|^2 - \|\vec{\mathbf{v}}\|^2.$$

(6) Show that there are no vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ such that $\|\vec{\mathbf{u}}\| = 1$, $\|\vec{\mathbf{v}}\| = 2$ and $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 3$.