

Homework 6: Due March 8th (Friday)

- (1) (a) Show that $\mathbb{R}^2 = \text{span} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$.
- (b) Show that $\mathbb{R}^3 = \text{span} \left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$.
- (2) For any of the sets below, decide if they are linearly independent, or linearly dependent, and state why. If they are linearly dependent, find a dependence relationship among the vectors.

(a) $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$

- (3) Canada's health guide recommends $625\mu\text{g}$ of vitamin A, $10\mu\text{g}$ of vitamin D, $12000\mu\text{g}$ of vitamin E, and $120\mu\text{g}$ of vitamin K per day, for average male university student. In university of Hard Knocks, the cafeteria offers three food items per day: special 1, special 2, and special 3. Here is the table of their vitamin content (all units are in μg):

	Special 1	Special 2	Special 3
Vitamin A	100	125	75
Vitamin D	1	2	2
Vitamin E	2000	2500	1000
Vitamin K	20	30	0

Can an average male university student here have a healthy diet? (Assume that a healthy diet means getting exactly the right amount of these four vitamins.)

- (4) Prove that \vec{u} , \vec{v} , and \vec{w} are all in the $\text{span}(\vec{u}, \vec{u} + \vec{v}, \vec{u} + \vec{v} + \vec{w})$.
- (5) Show that regardless of what vectors \vec{u} , \vec{v} , and \vec{w} are, the three vectors $\vec{u} - \vec{v}$, $\vec{v} - \vec{w}$, and $\vec{w} - \vec{u}$ are linearly dependent.