

Homework 7: Due March 15th (Friday)

(1) Let

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, D = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}, E = [4 \ 2], F = [-1 \ 2].$$

Compute the indicated matrices (if possible).

- (a) $A + 2D$
 - (b) $B - C$
 - (c) AB
 - (d) $D + BC$
 - (e) $E(AF)$
 - (f) $F(DF)$
 - (g) $AD - DA$.
- (2) Give an example of a nonzero 2×2 matrix A such that $A^2 = 0$.
- (3) (a) Let

$$E_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Calculate E_1A , E_2A , and E_3A .

(b) Find a 2×2 matrix so that

$$EA = \begin{bmatrix} a + 2c & b + 2d \\ c & d \end{bmatrix},$$

that is EA is the matrix you get by applying $R_1 + 2R_2$ to A .

(4) Find a 3×3 matrix C so that

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Hint: Find C column by column)

(5) Show that vectors v_1, v_2, v_3 in \mathbb{R}^4 can never span \mathbb{R}^4 .