

Homework 8: Due March 28th (**Thursday!**)

For all of the questions below, let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 3 & 5 \end{bmatrix}$$

and let

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

- (1) Find vector  $\vec{x}_1$  such that  $A\vec{x}_1 = \vec{e}_1$ .
- (2) Find vector  $\vec{x}_2$  such that  $A\vec{x}_2 = \vec{e}_2$ .
- (3) Find vector  $\vec{x}_3$  such that  $A\vec{x}_3 = \vec{e}_3$ .
- (4) Find  $A^{-1}$ .

- (5) Let  $\vec{b} = b_1\vec{e}_1 + b_2\vec{e}_2 + b_3\vec{e}_3 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ . Show that

$$A(b_1\vec{x}_1 + b_2\vec{x}_2 + b_3\vec{x}_3) = \vec{b}.$$

- (6) Let  $\vec{x}$  and  $\vec{b}$  be vectors such that  $A\vec{x} = \vec{b}$ . Show that  $A^{-1}\vec{b} = \vec{x}$ .
- (7) (**Bonus**) Let  $M$  be an  $m \times n$  matrix. Show that there is a  $n \times m$  matrix  $N$  such that  $MN = I_m$  if and only if the column vectors of  $M$  span  $\mathbb{R}^m$ .