

Homework 9: Due April 5th (Friday)

- (1) Calculate the rank of

$$\begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}.$$

(Rank of a matrix is the number of pivot elements in the echelon form of the matrix.)

- (2) If A is a 3×5 matrix, explain why the columns of A must be linearly dependent.
- (3) Give an example of invertible matrices A and B , so that

$$(A + B)^{-1} \neq A^{-1} + B^{-1}.$$

- (4) Find three 2×2 matrices satisfying $A^2 = A$.
- (5) Let A be an $n \times m$ matrix. Show that if $A\vec{x} = \vec{0}$ and $A\vec{y} = \vec{0}$, then $A\vec{u} = \vec{0}$, where \vec{u} is any linear combination of \vec{x} and \vec{y} .