

Sample Final

- (1) Solve the following system of equation:

$$\begin{aligned} 2a + 3b + 4c &= 5 \\ a + 2b + c &= 0 \\ 3a + b + 2c &= -4 \end{aligned}$$

- (2) Find k such that

$$\begin{aligned} 2x_2 + 3x_3 &= 8 \\ 2x_1 + 3x_2 + x_3 &= 5 \\ x_1 - x_2 - kx_3 &= -5 \end{aligned}$$

has no solutions.

- (3) Find projection of vector $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto $w = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

- (4) (a) Decide if the vectors

$$\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

span \mathbb{R}^3 , and if they are linearly dependent or independent.

- (b) Decide if the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3/7 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix},$$

span \mathbb{R}^4 , and if they are linearly dependent or independent.

- (5) Let

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, E = [4 \ 2], F = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

Compute the following matrices if possible. If not possible, say undefined.

- (a) $A + BC$
 (b) $E + 4F$
 (c) BA
 (d) EAF
 (e) $AD - DA$
- (6) (a) Find three by three matrices E_1, E_2 , and E_3 corresponding to the row reduction $R_2 + 3R_1, 2R_2$ and $R_1 \leftrightarrow R_3$.
 (b) Calculate $\det(E_1), \det(E_2)$, and $\det(E_3)$.
- (7) (a) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 2 \\ 1 & 3 & -2 \end{bmatrix}$$

(b) Find matrix B so that

$$AB = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(8) Calculate $\det(A)$ where

$$A = \begin{bmatrix} 1 & 5 & -2 \\ 2 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}.$$

- (9) (a) Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$. Find λ_1 and λ_2 , the eigenvalues of A .
(b) Find eigenvectors v_1 and v_2 of A corresponding λ_1 and λ_2 .
(c) Calculate A^5 .