MATH 1410
Solutions for the Sample Second Midterm

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(1) Solve the following systems of equations.

(a)
$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & 3x_3 & = & 9 \\ 2x_1 & - & x_2 & + & x_3 & = & 0 \\ 4x_1 & - & x_2 & + & x_3 & = & 4 \end{array}$$

Solution:

We do the usual thing: we put our right hands in, we take our right hands out, we put our right hands in, and we shake them all about…wait, that's the hokey pokey. I meant, we create and row reduce the associated augmented matrix:

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 2 & 0 & 0 & 4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 2 & -1 & 1 & 0 \\ 1 & 2 & -3 & 9 \end{array}\right]$$

$$\xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - R_1}} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & -1 & 1 & -4 \\ 0 & 2 & -3 & 7 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 2 & -3 & 7 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

(continued)

(continued)

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

$$\xrightarrow{-R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

This matrix is in RREF (reduced row echelon form). Writing down the system that it represents gives us the only solution for the original system:

$$\boxed{x_1 = 2,\ \ x_2 = 5,\ \ x_3 = 1.}$$

(b)
$$\begin{array}{ccccccccc} a & + & b & + & c & + & d & = & 4 \\ a & + & 2b & + & 3c & + & 4d & = & 10 \\ a & + & 3b & + & 6c & + & 10d & = & 20 \\ a & + & 4b & + & 10c & + & 20d & = & 35 \end{array}$$

<u>Solution</u>:

We create and reduce the associated augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 10 \\ 1 & 3 & 6 & 10 & 20 \\ 1 & 4 & 10 & 20 & 35 \end{array}\right]$$

$$\begin{array}{c} R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \\ \longrightarrow \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 6 \\ 0 & 2 & 5 & 9 & 16 \\ 0 & 3 & 9 & 19 & 31 \end{array}\right]$$

(continued)

(continued)

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 6 \\ 0 & 2 & 5 & 9 & 16 \\ 0 & 3 & 9 & 19 & 31 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ R_3 - 2R_2 \\ R_4 - 3R_2 \\ \longrightarrow \end{array} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & -2 \\ 0 & 1 & 2 & 3 & 6 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 3 & 10 & 13 \end{array}\right]$$

$$\begin{array}{c} R_1 + R_3 \\ R_2 - 2R_3 \\ R_4 - 3R_3 \\ \longrightarrow \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -3 & -2 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_4 \\ R_2 + 3R_4 \\ R_3 - 3R_4 \\ \longrightarrow \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right].$$

By observation, the only solution is $\boxed{a = 1,\ b = 1,\ c = 1,\ d = 1.}$

(2) Row reduce the following matrices to reduced row echelon form, and circle the pivot elements.

(a) $\begin{bmatrix} 4 & -2 & 1 & 0 \\ 4 & -2 & 0 & 1 \\ -2 & -1 & 4 & -2 \end{bmatrix}$

<u>Solution</u>:

I might not be allowed to use a calculator, so I'm going to avoid fractions as long as I can:

$$\begin{bmatrix} 4 & -2 & 1 & 0 \\ 4 & -2 & 0 & 1 \\ -2 & -1 & 4 & -2 \end{bmatrix}$$

$$\begin{array}{c} R_1 + 2R_3 \\ R_2 + 2R_3 \\ \longrightarrow \end{array} \begin{bmatrix} 0 & -4 & 9 & -4 \\ 0 & -4 & 8 & -3 \\ -2 & -1 & 4 & -2 \end{bmatrix}$$

(continued)

(continued)

$$= \begin{bmatrix} 0 & -4 & 9 & -4 \\ 0 & -4 & 8 & -3 \\ -2 & -1 & 4 & -2 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -2 & -1 & 4 & -2 \\ 0 & -4 & 8 & -3 \\ 0 & -4 & 9 & -4 \end{bmatrix}$$

$$\xrightarrow{-4R_1} \begin{bmatrix} 8 & 4 & -16 & 8 \\ 0 & -4 & 8 & -3 \\ 0 & -4 & 9 & -4 \end{bmatrix}$$

$$\xrightarrow{\substack{R_1+R_2 \\ R_3-R_2}} \begin{bmatrix} 8 & 0 & -8 & 5 \\ 0 & -4 & 8 & -3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_1+8R_3 \\ R_2-8R_3}} \begin{bmatrix} 8 & 0 & 0 & -3 \\ 0 & -4 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow{\substack{(1/8)R_1 \\ (-1/4)R_2}} \boxed{\begin{bmatrix} \textcircled{1} & 0 & 0 & -3/8 \\ 0 & \textcircled{1} & 0 & -5/4 \\ 0 & 0 & \textcircled{1} & -1 \end{bmatrix}.}$$

(b) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 3/7 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

<u>Solution</u>:

WHAT?! There's *already* a fraction! DIE, FRACTION, DIE!

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 3/7 & 0 \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{7}{3}R_3} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Whew! Now we can continue row reducing the matrix.

(continued)

(continued)

$$
\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}
$$

$$
\begin{array}{c} R_1 \leftrightarrow R_4 \\ R_2 \leftrightarrow R_3 \\ \longrightarrow \end{array}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}
$$

$$
\begin{array}{c} R_3 - R_1 \\ R_4 - R_1 \\ \longrightarrow \end{array}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}
$$

$$
\begin{array}{c} R_3 - R_2 \\ R_4 - 2R_2 \\ \longrightarrow \end{array}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{array}{c} R_3 \leftrightarrow R_4 \\ \longrightarrow \end{array}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}
$$

$$
\begin{array}{c} R_4 + 2R_3 \\ \longrightarrow \end{array}
\boxed{\begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \\ 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 \end{bmatrix}.}
$$

(3) (a) Decide if the vectors

$$\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

span $\mathbb{R}^3$, and if they are linearly dependent or independent.

<u>Solution</u>:

To make *both* decisions, we first create a matrix whose columns are the given vectors:

$$\begin{bmatrix} 4 & -2 & 1 & 0 \\ 4 & -2 & 0 & 1 \\ -2 & -1 & 4 & -2 \end{bmatrix}.$$

We then row reduce this matrix to reduced row echelon form...WAITAMINUTE! We *already* reduced this matrix in (2)(a)! The RREF that we got was

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & -3/8 \\ 0 & \textcircled{1} & 0 & -5/4 \\ 0 & 0 & \textcircled{1} & -1 \end{bmatrix}.$$

What a nice coinkydink! (◡̈) Anyways, now that we have the RREF, we can answer both questions.

First, the given vectors span the $\mathbb{R}^n$ to which they belong if and only if the RREF of the matrix in which they were the columns has a *leading one in every row.* In our case, the RREF has a leading one in each of its three rows, so the given vectors $\boxed{\textit{do} \text{ span } \mathbb{R}^3.}$

Second, the given vectors are linearly **in**dependent if and only if the RREF of the matrix in which they were the columns has a *leading one in every column.* In our case, the RREF does *not* have a leading one in its fourth column, so the given vectors are *not* linearly independent, which means that they are $\boxed{\text{linearly } \textbf{de}\text{pendent.}}$

(b) Decide if the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3/7 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

span $\mathbb{R}^4$, and if they are linearly dependent or independent.

Solution:

Again, we create a matrix whose columns are the given vectors:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -\mathbf{1} \\ 0 & 3/7 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Then we look at (2)(b) and notice that IT'S NOT THE SAME MATRIX! Master betrayed us! Wicked, tricksy, false! Anyways, let's row reduce this new matrix, precious:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 0 & 3/7 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\frac{7}{3}R_3} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\substack{R_1 \leftrightarrow R_4 \\ R_2 \leftrightarrow R_3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_3 - R_1 \\ R_4 - R_1}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_3 - R_2 \\ R_4 - 2R_2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(continued)

(continued)

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_4 - R_3} \boxed{\begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \\ 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 \end{bmatrix}}.$$

This matrix is in RREF, so we can make our decisions. To begin, the fourth row of this matrix does *not* contain a leading one, so the given vectors $\boxed{\text{do } \textit{not} \text{ span } \mathbb{R}^4.}$

Next, this matrix has a leading one in each of its three columns, so the given vectors *are* $\boxed{\text{linearly } \textbf{in}\text{dependent.}}$

(4) Let $A = \begin{bmatrix} 2 & 5 & 4 \\ 0 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$.

Calculate each of the following matrices if possible. If not, say undefined.

(a) $A + B$

Solution:

We notice that A is 2×3 while B is 3×2; since their sizes differ, their sum is $\boxed{\text{undefined.}}$

(b) $AB + C$

Solution:

Let us first ensure that the desired matrix is defined: A is 2×3 and B is 3×2 i.e. the inside numbers are both 3's, so AB is defined and its size is given by the outside numbers: 2×2. This matches the size of C, so $AB + C$ is also defined.

To compute the desired matrix, we begin with the product AB. We've seen how to multiply matrices using linear combinations of column vectors, but there is a more compact[†] method:

Matrix Multiplication 2.0: Let A be an $m \times n$ matrix and B be an $n \times p$ matrix, so that AB is defined and has size $m \times p$. To use this technique, we consider both the *rows* of A and the *columns* of B to be vectors from $\mathbb{R}^n$. Then, the entry in row i and column j of AB (also called its (i, j)-entry) is the *dot product* of row i in A with column j in B.

Let's see this in action: $AB = \left[\begin{array}{ccc} 2 & 5 & 4 \\ \hline 0 & 3 & -1 \end{array}\right] \left[\begin{array}{c|c} -2 & 0 \\ 1 & 1 \\ 0 & 2 \end{array}\right]$

$$= \begin{bmatrix} (2)(-2)+(5)(1)+(4)(0) & (2)(0)+(5)(1)+(4)(2) \\ (0)(-2)+(3)(1)+(-1)(0) & (0)(0)+(3)(1)+(-1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} -4+5+0 & 0+5+8 \\ 0+3-0 & 0+3-2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 13 \\ 3 & 1 \end{bmatrix}.}$$

(c) $BA + C$

Solution:

Since B is 3×2 and A is 2×3, BA is defined and has size 3×3, which does *not* match the size of C. Therefore, $BA + C$ is $\boxed{\text{undefined.}}$

[†]It requires the same amount of arithmetic (we do the same number of multiplications and the same number of additions/subtractions) but *far* less writing.

(d) CAB

Solution:

Hmmm...are we supposed to multiply CA first and then multiply our result on the left of B, or do we multiply AB first and then multiply C on the left of our result?

It doesn't matter! The two procedures described above will produce the *same* answer; that is, $(CA)B = C(AB)$. Actually, the latter one is preferable because we've *already* computed AB:

$$CAB = C(AB) = \left[\begin{array}{cc} 2 & 0 \\ \hline 1 & 2 \end{array}\right]\left[\begin{array}{c|c} 1 & 13 \\ 3 & 1 \end{array}\right] = \begin{bmatrix} (2)(1)+(0)(3) & (2)(13)+(0)(1) \\ (1)(1)+(2)(3) & (1)(13)+(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 26+0 \\ 1+6 & 13+2 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 26 \\ 7 & 15 \end{bmatrix}.}$$

(e) $B + A^{-1}$

Solution:

Let A and B be arbitrary matrices (not necessarily the ones defined in this question). If $AB = BA = I$, we call B the *inverse* of A and give it the label A^{-1}.

Now say that the $p \times q$ matrix B is the inverse of the $m \times n$ matrix A. Then, AB is defined, so $n = p$; moreover, AB is $m \times q$. Also, BA is defined, so $q = m$; moreover, BA is $p \times n$. Lastly, $AB = BA$, so $m = p$ and $q = n$; this means that $m = n = p = q$ i.e. A and B are *necessarily* square matrices of the same size.

Back to the question at hand: the given matrix A is *not* square (it has more columns than rows), so its inverse, A^{-1}, does *not* exist. Hence, $B + A^{-1}$ is $\boxed{\text{undefined.}}$

(f) C^{-1}

Solution:

C is square, so it *could* have an inverse D; no promises, though! If it existed, D would be the same size as C and satisfy $CD = DC = I$. Of course, that equation can only be satisfied if CD is the 2×2 identity matrix, so let us find entries a, b, c, and d in D such that $CD = I_2$, or

$$\left[\begin{array}{cc} 2 & 0 \\ \hline 1 & 2 \end{array}\right]\left[\begin{array}{c|c} a & b \\ c & d \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \implies \left[\begin{array}{cc} 2a & 2b \\ a+2c & b+2d \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right].$$

These two matrices can only be equal if their corresponding entries are equal. Comparing their $(1,1)$-entries, we see that $2a = 1$, so $a = 1/2$. Equating their $(1,2)$-entries, we find that $b = 0$. Continuing this for the remaining corresponding entries, we get

$$a + 2c = 0 \implies c = -\frac{a}{2} = -\frac{1}{4} \quad \text{and} \quad b + 2d = 1 \implies d = \frac{1-b}{2} = \frac{1}{2}.$$

We hypothesize that $D = \left[\begin{array}{cc} 1/2 & 0 \\ -1/4 & 1/2 \end{array}\right]$. To confirm that it is the inverse of C, we compute

$$DC = \left[\begin{array}{cc} 1/2 & 0 \\ \hline -1/4 & 1/2 \end{array}\right]\left[\begin{array}{c|c} 2 & 0 \\ 1 & 2 \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] = I.$$

D is indeed the inverse of C. Ergo, $C^{-1} = \boxed{\left[\begin{array}{cc} 1/2 & 0 \\ -1/4 & 1/2 \end{array}\right].}$

(5) Let $A = \left[\begin{array}{cc} 1 & x \\ 0 & 1 \end{array}\right]$, and assume that $A^5 = \left[\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array}\right]$.

Solve for x.

Solution:

There are two ways to compute A^5.

Method 1: Brute force!

$$A^2 = AA = \left[\begin{array}{cc} 1 & x \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} 1 & x \\ 0 & 1 \end{array}\right] = \left[\begin{array}{cc} 1 & 2x \\ 0 & 1 \end{array}\right].$$

(continued)

(continued) $A^3 = AA^2 = \left[\begin{array}{cc} 1 & x \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} 1 & 2x \\ 0 & 1 \end{array}\right] = \begin{bmatrix} 1 & 3x \\ 0 & 1 \end{bmatrix}.$

$A^4 = AA^3 = \left[\begin{array}{cc} 1 & x \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} 1 & 3x \\ 0 & 1 \end{array}\right] = \begin{bmatrix} 1 & 4x \\ 0 & 1 \end{bmatrix}.$

$A^5 = AA^4 = \left[\begin{array}{cc} 1 & x \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} 1 & 4x \\ 0 & 1 \end{array}\right] = \begin{bmatrix} 1 & 5x \\ 0 & 1 \end{bmatrix}.$

Method 2: Elementary matrices

Since A may be created by performing $\mathrm{R}_1 + x\,\mathrm{R}_2$ on the 2×2 identity matrix, it is an elementary matrix that represents this operation i.e. multiplying another matrix on the left by A performs $\mathrm{R}_1 + x\,\mathrm{R}_2$ on that other matrix. Left-multiplying by A^5 means left-multiplying by A five times in sequence, which means that the operation is performed five times in sequence. Consequently,

$$A^5 = A^5 I = A^5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5x \\ 0 & 1 \end{bmatrix}.$$

Whatever way we compute it, A^5 must be equal to $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$\implies \begin{bmatrix} 1 & 5x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \implies 5x = 1 \implies x = \boxed{1/5.}$$

(6) Find 2×2 matrices A and B so that $AB \neq BA$.

Solution:

Would you believe? If you randomly select entries, you are *more* likely to make $AB \neq BA$ than you are to make $AB = BA$!

Here goes: let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$.

Then, $AB = \left[\begin{array}{cc} 1 & 2 \\ \hline 3 & 4 \end{array}\right]\left[\begin{array}{c|c} 5 & 6 \\ 7 & 8 \end{array}\right] = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix},$

while $BA = \left[\begin{array}{cc} 5 & 6 \\ \hline 7 & 8 \end{array}\right]\left[\begin{array}{c|c} 1 & 2 \\ 3 & 4 \end{array}\right] = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix},$ so $AB \neq BA$. It worked!