

Sample Midterm

- (1) Find the roots of the following polynomials:

(a) $(x - 1)(x^2 + 5x + 6)$

(b) $(x - 4)^2 - 3$

- (2) Solve the following systems of equations:

(a)

$$x + y + z = 7$$

$$2x + y - 3z = 7$$

$$x + 2y - 2z = 6$$

(b)

$$2a + 3b + 4c = 5$$

$$3a + 2b + 3c = 2$$

$$5b + 6c = 11$$

(c)

$$a + b + c + d = 9$$

$$2a + 3b + c + d = 5$$

$$a + 2b = 1$$

- (3) Give an example of a 3-equations and 3-unknowns system of linear equations that has no solutions.

- (4) Let $\vec{\mathbf{u}} = [1, 2, -3]$ and $\vec{\mathbf{v}} = [-3, 5, 2]$.

(a) Calculate $\|\vec{\mathbf{u}}\|$

(b) Calculate $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$

(c) Calculate the projection of vector $\vec{\mathbf{u}}$ onto vector $\vec{\mathbf{v}}$.

(d) Show that the vector $[-1, 9, -4]$ is a linear combination of $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

- (5) Give an example of three vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{w}}$ such that $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{u}} \cdot \vec{\mathbf{w}}$, but $\vec{\mathbf{v}} \neq \vec{\mathbf{w}}$.

- (6) Show that

$$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}}) = \|\vec{\mathbf{u}}\|^2 - \|\vec{\mathbf{v}}\|^2.$$