

Sample Second Midterm

(1) Solve the following systems of equations.

(a)

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 9, \\2x_2 - x_2 + x_3 &= 0, \\4x_1 - x_2 + x_3 &= 4\end{aligned}$$

(b)

$$\begin{aligned}a + b + c + d &= 4 \\a + 2b + 3c + 4d &= 10 \\a + 3b + 6c + 10d &= 20 \\a + 4b + 10c + 20d &= 35\end{aligned}$$

(2) Row reduce the following matrices to reduced row echelon form, and circle the pivot elements.

(a)

$$\begin{bmatrix} 4 & -2 & 1 & 0 \\ 4 & -2 & 0 & 1 \\ -2 & -1 & 4 & -2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 3/7 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(3) (a) Decide if the vectors

$$\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

span  $\mathbb{R}^3$ , and if they are linearly dependent or independent.

(b) Decide if the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3/7 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix},$$

span  $\mathbb{R}^4$ , and if they are linearly dependent or independent.

(4) Let

$$A = \begin{bmatrix} 2 & 5 & 4 \\ 0 & 3 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}.$$

Calculate each of the following matrices if possible. If not, say undefined.

- (a)  $A + B$
- (b)  $AB + C$
- (c)  $BA + C$
- (d)  $CAB$

(e)  $B + A^{-1}$

(f)  $C^{-1}$

(5) Let  $A = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$ , and assume that

$$A^5 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Solve for  $x$ .

(6) Find  $2 \times 2$  matrices  $A$  and  $B$  so that  $AB \neq BA$ .